

## MATH 13 FALL 2019 HOMEWORK 1

**Due: Thursday, October 17, 2019** Please turn in at the discussion.

If you work in a group of two please turn in only one paper, but put both names and Student ID's on the paper. If you work alone, turn in a paper with your name and Student ID only.

Student name/id (include all students in the group):

### IMPORTANT INSTRUCTIONS:

- It is crucial that you write your arguments and explanations clearly and that each argument clearly shows how you arrive at the conclusions from the assumptions. This is the point of homeworks – to practice understanding of the material, proofwriting, and the ability to express your understanding.
- Try to write your arguments as efficiently as possible, meaning that you should judge what to write and what not. Writing irrelevant text makes the argument confused and difficult to understand. Leaving out important points makes the argument incomplete. So you need to judge what is relevant and what not. There is no recipe for this; the ability to recognize what to write and what not is a skill which needs to be developed, and this course (= Math 13) is intended to help you with this.
- When preparing the homeworks, please follow the Rules for homeworks on the course website under Course information and policies.

**1. (5pt)** Peter, Quin and Robert are discussing whether they order pizza for lunch. Here are the preferences:

- Peter will order pizza precisely when both Quin and Robert order pizza.
- Quin will order pizza if Robert does not order pizza.
- Robert will order pizza only if Peter and Quin will make opposite decisions regarding ordering pizza

Who will order pizza?

**Remark.** It is o.k. to make use of truth tables.

**2. (5pt)** We consider sets  $A, B \subseteq U$ , where  $U$  is a background set. So complements are formed with respect to  $U$ . In each of the following cases decide whether the equality or inclusion in question is true. For this, look at the elements of sets considered, and **provide explanations** to your answers.

- (a) **(1pt)**  $A \triangle A = \emptyset$ .

- (b) (1pt)  $A \setminus A^c = A$ .
- (c) (1pt)  $A \setminus B = A \setminus (A \cap B)$ .
- (d) (1pt)  $A \Delta A^c = U$ .
- (e) (1pt)  $A \Delta B \subseteq A \cup B$ .

It is crucial that you provide explanation. Answers without explanation do not count, that is, are worth 0 points.

3. (5pt) Translate the following statements into human language.

- (a) (1pt)  $x \in \mathbb{Z} \wedge (\exists y \in \mathbb{Z})(x = 2 \cdot y)$ .
- (b) (1pt)  $(\exists y \in \mathbb{Z})(\exists z \in \mathbb{Z})(x = y^2 + z^2)$ .
- (c) (1pt)  $(\exists x \in A)(\exists y \in A)(\exists z \in A)(x \neq y \wedge x \neq z \wedge y \neq z)$ .
- (d) (1pt)  $x \in \mathbb{R} \wedge (\forall y \in \mathbb{R})(x < y^2)$ .
- (e) (1pt)  $(\forall x \in \mathbb{R})[x \geq 0 \implies (\exists y \in \mathbb{R})(x = y^2)]$ .

4. (5pt) Translate the following statements into symbolic language.

- (a) (1pt) There exist at least three even integers.
- (b) (1pt) The cube of any even number is divisible by 8.
- (c) (1pt) The square of any real number is nonnegative.
- (d) (1pt) Between any two distinct real numbers there is a rational number.
- (e) (1pt) The equation  $x^2 + x + 1$  has two distinct complex roots.