MATH 13 FALL 2019 HOMEWORK 2

Due: Thursday, October 24, 2019 Please turn in at the discussion.

If you work in a group of two please turn in only one paper, but put both names and Student ID's on the paper. If you work alone, turn in a paper with your name and Student ID only.

Student name/id (include all students in the group):

IMPORTANT INSTRUCTIONS:

- It is crucial that you write your arguments and explanations clearly and that each argument clearly shows how you arrive at the conclusions from the assumptions. This is the point of homeworks – to practice understanding of the material, proofwriting, and the ability to express your understanding.
- Try to write your arguments as efficiently as possible, meaning that you should judge what to write and what not. Writing irrelevant text makes the argument confused and difficult to understand. Leaving out important points makes the argument incomplete. So you need to judge what is relevant and what not. There is no recipe for this; the ability to recognize what to write and what not is a skill which needs to be developed, and this course (= Math 13) is intended to help you with this.
- When preparing the homeworks, please follow the **Rules for homeworks** on the course website under **Course information and policies**.

ALSO IMPORTANT: In arguments and proofs you are writing, **never** use symbols $\therefore$ and $\because$ but instead, write everything as a text.

Also avoid using the symbol $\nexists$.

1. (5pt) Give a direct proof of the following: If x, y are integers not divisible by 3 such that their remainders after dividing by 3 are distinct then $x^3 + y^3$ is divisible by 3.

Remark. Recall: To prove that a number z is divisible by 3 you need to produce an integer k such that $z = 3 \cdot k$.

2. (5pt) Assume A, B are subsets of U. The complements A^c, B^c are computed with respect to U. Give an indirect proof (in other words, prove the contrapositive) of the following: If $A^c \subseteq B^c$ then $B \subseteq A$.

3. (5pt) Give a proof by cases of the following: If x is an integer not divisible by 5 then x^4 gives remainder 1 after dividing by 5.

Remark. A remark similar to that at the end of Problem 1, adapted to the current context, applies.

4. (5pt) Give a proof by contradiction of the following: $\sqrt{6}$ is not a rational number.

Try to generalize the proof that $\sqrt{2}$ is not a rational number, which was discussed in the lecture/discussion. **Prove all relevant intermediate statements you are using in your argument.**