MATH 13 FALL 2019 HOMEWORK 4

Due: Thursday, November 14, 2019 Please turn in at the discussion.

If you work in a group of two please turn in only one paper, but put both names and Student ID's on the paper. If you work alone, turn in a paper with your name and Student ID only.

Student name/id (include all students in the group):

IMPORTANT INSTRUCTIONS:

- It is crucial that you write your arguments and explanations clearly and that each argument clearly shows how you arrive at the conclusions from the assumptions. This is the point of homeworks – to practice understanding of the material, proofwriting, and the ability to express your understanding.
- Try to write your arguments as efficiently as possible, meaning that you should judge what to write and what not. Writing irrelevant text makes the argument confused and difficult to understand. Leaving out important points makes the argument incomplete. So you need to judge what is relevant and what not. There is no recipe for this; the ability to recognize what to write and what not is a skill which needs to be developed, and this course (= Math 13) is intended to help you with this.
- When preparing the homeworks, please follow the **Rules for homeworks** on the course website under **Course information and policies**.

ALSO IMPORTANT: In arguments and proofs you are writing, **never** use symbols $\therefore$ and $\because$ but instead, write everything as a text.

Also avoid using the symbol $\nexists$.

1. (5pt) In the class we gave a proof that if a is an integer then there are **at least one** integer q and **at least one** integer r such that

(i) $0 \leq r < 5$, and
(ii) $a = 5 \cdot q + r$

We then formulated a general result with an integer $b > 1$ in place of 5.

Formulate this general result with b in place of 5 again, and then give a proof of the general result, which mimics the proof with number 5 in the lecture. Write the proof carefully.

2. (5pt) Assume $a, b, n \in \mathbb{Z}$ and $n \neq 0$. Prove that the following are equivalent:

(a) a, b give the same remainder when dividing by n.
(b) $n \,|(a-b)$.

3. (5pt) Assume $a, a', b, b', n \in \mathbb{Z}$ are such that

(i) $n \neq 0$.
(ii) $a \equiv a' \mod n$.
(iii) $b \equiv b' \mod n$.

In the lecture we gave a proof, using the equivalence from the previous exercise, that $a'b' \equiv ab \mod n$.

(a) Prove that $a' + b' \equiv a + b \mod n$.
(b) Prove by induction on k that $a^k \equiv (a')^k \mod n$ for all $k \in \mathbb{N}$. For induction step quote, but **not** prove, the above result from the lecture concerning multiplication.

4. (5pt) Use the previous exercise to calculate the remainder of

$$(5 \cdot 4^{100} + 8 \cdot 7^{201})^{99}$$

after dividing by 9.