MATH 13 FALL 2019 MIDTERM PRACTICE PROBLEMS

Below are some sample problems similar to those which will be given on the midterm.

1. Here we consider sets $A, B, C \subseteq U$ where U is a universal set.

(a) If $A \subseteq B$ determine the relationship between A^c and B^c.
(b) If $A \triangle B \subseteq C$ determine the relationship between $A \setminus C$ and $A \cap B$.
(c) Determine $\mathcal{P}(A) \cap \mathcal{P}(A^c)$.
(d) Is $\mathcal{P}(A) \cup \mathcal{P}(A^c) = \mathcal{P}(U)$?
(e) Is $\mathcal{P}(A) \setminus \mathcal{P}(B) \subseteq \mathcal{P}(A \setminus B)$?

2. Express the following statements in the symbolic language using symbols as instructed in Problem group 1 and then negate the statement. Use only symbols $=, <, \leq$ and arithmetic operations like $+, -, \cdot$ and similar. In (e) consider a set $A \subseteq \mathbb{Q}$.

(a) Every positive integer can be expressed as a sum of three squares of integers.
(b) For every real number r the equation $8x + r = 0$ has precisely one real solution.
(c) There are at least three different integers in the interval $(-2, 2)$.
(d) For every real number x larger than 1 the value $1/x$ is smaller than 1 but larger than 0.
(e) Between any two distinct integers there is an element of A.

3. Prove the following statements. In each case choose a suitable type of proof.

(a) If a, b are odd integers then $a^2 - b^2$ is divisible by 4.
(b) If p is an odd integer then the equation $x^3 + x + p = 0$ does not have an integer solution.
(c) If a, b are integers divisible by an integer n then the product $a \cdot b$ is divisible by n^2.
(d) If a, b, c, d are four consecutive integers then the sum $a + b + c + d$ is even.
(e) The product of three consecutive integers is always divisible by 6.

4. Prove the following by induction on n.

(a) For each $n > 0$, $1^3 + 2^3 + \cdot + n^3 = \frac{1}{2}n^2(n+1)^2$.
(b) For each $n > 0$, $17^n - 13^n$ is divisible by 4.
(c) For every $n > 0$, $1 + a + a^2 + \cdot + a^n = (a^{n+1} - 1)/(a - 1)$.
(d) For every $n > 0$, $a^n - b^n = (a-b)(a^{n-1} + a^{n-1}b + \cdots + a^{n-k}b^k + \cdots + b^{n-1})$. It would be nicer to use the "Σ"-notation:

$$a^n - b^n = (a - b) \cdot \sum_{k=0}^{n-1} a^{n-1-k} b^k$$

(e) For every $n > 1$, $(2n + 1)!$ is divisible by $2^n \cdot n!$.