GRADED HOMEWORK 2

1. (2pt) Prove that $A \cap (B - C) = (A \cap B) - C$ for any sets A, B, C. This is a straightforward verification.

2. (4pt) Prove that

$$\mathcal{P}(A \cup B) = \{X \cup Y \mid X \in \mathcal{P}(A) \ \&\ Y \in \mathcal{P}(B)\}.$$

Hint. Use the extensionality principle. For $\subseteq$, pick any $Z \in \mathcal{P}(A \cup B)$ and show it can be expressed in the form $X \cup Y$ where $X \in \mathcal{P}(A)$ and $Y \in \mathcal{P}(B)$. The inclusion $\supseteq$ is easy.

3. (2pt) For each $p \in \mathbb{Z}$ and $n \in \mathbb{N}^+$ let

$$A_{p,n} = (p - \frac{1}{n}, p + \frac{1}{n}) = \{x \in \mathbb{R} \mid p - \frac{1}{n} < x < p + \frac{1}{n}\}.$$

For each $n \in \mathbb{N}^+$ let

$$A_n = \bigcup_{p \in \mathbb{Z}} A_{p,n}.$$

(Draw the picture). Prove that $\bigcap_{n \in \mathbb{N}^+} A_n = \mathbb{Z}$. Give a rigorous proof.

Hint. Again use extensionality. For $\supseteq$, show that every element of $\mathbb{Z}$ is in the intersection; this is easy. For $\subseteq$, show that any real number which is not an integer is outside some set A_n.

4. (2pt) Let $\{X_i \mid i \in I\}$ and $\{Y_j \mid j \in J\}$ be two indexed systems which satisfy the following: For every $i \in I$ there is some $j \in J$ such that $X_i \subseteq Y_j$. Symbolically:

$$i \in I \implies (\exists j \in J)(X_i \subseteq Y_j).$$

(Try to draw a picture!) Prove:

$$\bigcup_{i \in I} X_i \subseteq \bigcup_{j \in J} Y_j.$$

Hint. Pick an arbitrary element x in the union of all X_i. Use the definition of union and the fact that $X_i \subseteq Y_j$ for a suitable j to show that x is in the union of all Y_j.