MIDTERM-SAMPLE

1. (5pt) Prove that $X-(A\cap B)=(X-A)\cup(X-B)$ for any sets A, B, X.

Hint. This is a straightforward verification; use the extensionality principle.

2. (5pt) For each $n\in\mathbb{N}^+$ consider the open interval

$$I_n=(-\frac{1}{n},\frac{3n+4}{n+1}).$$

Give a rigorous proof that

$$\bigcap_{n\in\mathbb{N}^+} I_n=[0,3].$$

Hint. Use the extensionality principle in the straightforward way. Recall that you have to prove the two parts: First, that the closed interval $[0,3]$ is contained in the intersection and second, that any $x\notin[0,3]$ is not in the intersection.

3. (5pt) Prove by induction on n that for every integer $n>0$ we have

$$\frac{1}{1\cdot 3}+\frac{1}{3\cdot 5}+\frac{1}{5\cdot 7}+\cdot+\frac{1}{(2n-1)\cdot(2n+1)}=\frac{n}{2n+1}$$

4. (5pt) Prove by induction on n that for all $n\in\mathbb{N}$ the number

$$5\cdot 13^n+7\cdot 19^n$$

is divisible by 6.