MATH 150 HOMEWORK 2

DUE: Wednesday, October 24
Student Name/Id # (Include all students in the group):

IMPORTANT INSTRUCTION: It is crucial that you clearly state justifications for all your conclusions. This is the point of the homeworks – to practice understanding of the material and the ability to express your understanding.

1. (5pt) Consider the following situation: In a production line, a belt is carrying parts. There are three sensors, A, B and C that are watching if the distances of the parts on the production line are accurate (that is, the distances are not too large or too small). The rules are:

(a) There is a part at position A if and only if there is a part (a different one) at position C.
(b) There is a part at position A if and only if there is no part at position B.

If there is a part at any of the positions A, B or C then the corresponding sensor sends signal 1. If there is no part at the position, the corresponding sensor sends signal 0.

Task: Using units NAND build a device which will send signal 1 if both (a),(b) are satisfied, and will send signal 0 if one of (a),(b) is violated. Thus, signal 0 is considered an alert.

2. Recall the definition of tautological implication $\models$.

(a) (1pt) Let $A_1, \ldots, A_n$ and B be sentential letters. Decide if the following is true:

$$\{A_1 \to B, \ldots, A_n \to B\} \models (A_1 \vee \cdots \vee A_n) \to B.$$

If this is true, prove it. If false, disprove.

(b) (2pt) Consider a set of formulas Σ and two formulas φ and ψ. Prove:

$$\Sigma \cup \{\varphi\} \models \psi \quad \text{if and only if} \quad \Sigma \models \varphi \to \psi$$

Let me stress the meaning of "if and only if": You need to prove two things: If the left side holds then the right side holds, and vice versa: If the right side holds then also the left side holds.

(c) (2pt) Again consider a set of formulas Σ and a formula φ. Prove that $\Sigma \models (\varphi \wedge \neg\varphi)$ if and only if Σ is not finitely satisfiable. Here "if and only if" has the same meaning as in question (b) above.

3. Given are the following sets of sentential connectives.

(a) (2pt) $\{\mathsf{NOR}\}$, where recall that NOR is a binary connective defined by $\mathsf{NOR}(A, B) \equiv \neg(A \vee B)$.

(b) (3pt) $\{\leftrightarrow, \vee, \wedge\}$.

Decide if the set of connectives is complete. If complete, prove its completeness. If incomplete, prove the incompleteness. You may use the fact that $\{\neg, \vee\}$ and $\{\neg, \wedge\}$ are complete.