MATH 150 HOMEWORK 3

DUE: Friday, November 2
Student Name/Id # (Include all students in the group):

IMPORTANT INSTRUCTION: It is crucial that you clearly state justifications for all your conclusions. This is the point of the homeworks – to practice understanding of the material and the ability to express your understanding.

1. (5pt) For each of the following structures find a language for the respective structure and describe the interpretations of symbols of your language. Recall that for each function and relational symbol you need to specify the arity.

(a) **(1pt)** The domain of the structure is the set of all integers $\mathbb{Z}$. There are two specified objects in the domain, namely 0 and 1. The structure has usual operations of addition and multiplication. Additionally, there are the following functions: exponentiation $n \mapsto 2^{n^2}$, the function which to each two points assigns their distance, and a function that to each tuple of length five assigns the largest element.

(b) **(2pt)** The domain of the structure is the set of all real numbers $\mathbb{R}$. The structure has functions that to each finite set of real numbers assigns the mean value. Also the structure has relations that for each finite tuple of real numbers indicate whether the distances between the adjacent numbers are all the same.

(c) **(2pt)** The domain of the structure is the set of all lines in the Euclidean plane. The structure has a relation "line l is parallel with line l'". Additionally, the structure has relations that for each triple of lines carry the information about how many lines in the triple are parallel – make this precise.

2. 5pt Given is a structure $\mathcal{M}$ characterized as follows. The domain M of the structure $\mathcal{M}$ is the set of all people. The language has one constant symbol p, two unary function symbols F and G and one binary function symbol C. The interpretation of the constant symbol p is:

$$p^{\mathcal{M}} = \text{ the president of the people in } M.$$

The interpretations $F^{\mathcal{M}}, G^{\mathcal{M}}$ and $C^{\mathcal{M}}$ of these symbols are two unary functions

$$F^{\mathcal{M}} : M \to M \quad \text{and} \quad G^{\mathcal{M}} : M \to M$$

and a binary function

$$C^{\mathcal{M}} : M \times M \to M$$

with the following interpretations: For each $m \in M$,

$$
\begin{aligned}
F^{\mathcal{M}}(m) &= \text{ the mother of } m, \\
G^{\mathcal{M}}(m) &= \text{ the father of } m,,
\end{aligned}
$$

and for each pair $(m, n) \in M \times M$,

$$C^{\mathcal{M}}(m, n) = \begin{cases} \bullet \text{ the oldest child of the couple } (m, n) & \text{if the couple } (m, n) \text{ has a child} \\ \bullet \text{ the oldest child among all children of } m, n & \text{if the couple}(m, n) \text{ does not have a child but at least one of } m, \; n \text{ does have a child} \\ \text{the president} & \text{if } m, n \text{ do not have a child} \end{cases}$$

For each of the following situations write down the term describing the following persons.

   (a) **(1pt)** The great-grandmother of u from the father's and grandfather's side.

   (b) **(1pt)** The grandfather from the mother's side of the grandmother from the mother's side of u.

   (c) **(1pt)** The oldest sibling of u, granting that u has a sibling.

   (d) **(1pt)** The oldest uncle/aunt of u from mother's side, granting u has one.

   (e) **(1pt)** The husband of the president's daughter, granting that the president has only one child, this child is a daughter, and she has a child with her husband.

3. (5pt) Let $\mathcal{L}$ be a language.

   (a) **(2pt)** Give an inductive definition of the number of function symbols in a term.

   (b) **(3pt)** Prove by induction on complexity that the number of parentheses in a term is twice the number of function symbols that appear in it.

     Here we count occurrences of the symbols: This means that each symbol in question is counted as many times as many occurrences it has in the term.