MATH 150 FALL 2012

LIST OF LANGUAGES AND ASSOCIATED STRUCTURES

1. Language of Arithmetic. $\mathcal{L} = \{\dot{0}, \dot{S}, \dot{+}, \dot{\times}, \dot{<}\}$.

- Constant symbols: $\dot{0}$.
- Function symbols:
  - Unary function symbols: $\dot{S}$.
  - Binary functions symbols: $\dot{+}$, $\dot{\times}$.
- Relational symbols: Binary relational symbol $\dot{<}$.

Associated structures:

(a) **Standard model of arithmetic.** $\mathbb{N} = (N, 0, S, +, \cdot, <)$. The domain of $\mathbb{N}$ is

$$N = \text{the set of all nonnegative integers}$$

Interpretations of $\mathcal{L}$-symbols:
- $\dot{0}^{\mathbb{N}} = 0 =$ number zero.
- $\dot{S}^{\mathbb{N}} =$ the "successor" function S, that is, $S(n) =$ the least number larger than n; this number is called the **successor** of n.
- $\dot{+}^{\mathbb{N}} =$ the usual addition of numbers $+$, that is, $\dot{+}^{\mathbb{N}}(m, n) = m + n$.
- $\dot{\times}^{\mathbb{N}} =$ the usual multiplication of numbers $\cdot$, that is, $\dot{\times}^{\mathbb{N}}(m, n) = m \cdot n$.
- $\dot{<}^{\mathbb{N}} =$ the usual ordering of numbers, that is, $(m, n) \in \dot{<}^{\mathbb{N}}$ if and only if $m < n$.

2. Language of Group Theory, also briefly "**language of groups**". $\mathcal{L} = \{\circ, e\}$.

- Constant symbols: e.
- Function symbols: Binary function symbol $\circ$.

Associated structures:

(a) **The additive group of integers** $\mathbb{Z} = (Z, +, 0)$. The domain of $\mathbb{Z}$ is

$$Z = \text{the set of all integers}$$

Interpretations of $\mathcal{L}$-symbols:
- $e^{\mathbb{Z}} = 0 =$ number 0.
- $\circ^{\mathbb{Z}} =$ the usual addition of integers, i.e. $\circ^{\mathbb{Z}}(m, n) = m + n$.

Similarly we define interpretations $\mathbb{Q} = (Q, +, 0)$, $\mathbb{R} = (R, +, 0)$ and $\mathbb{C} = (C, +, 0)$ where Q, R and C are sets of all rational, real, and complex numbers, respectively.

(b) **The multiplicative group of rational numbers** $\mathbb{Q}^{\times} = (Q^{\times}, \cdot, 1)$**.** The domain of $\mathbb{Q}^{\times}$ is

$$Q^{\times} = \text{the set of all rational numbers without 0.}$$

Interpretations of $\mathcal{L}$-symbols:
- $e^{\mathbb{Q}^{\times}} = 1 =$ number 1.
- $\circ^{\mathbb{Q}^{\times}} =$ the usual addition of integers, i.e. $\circ^{\mathbb{Q}^{\times}}(m, n) = m \cdot n$.

Similarly we define interpretations $\mathbb{R}^{\times} = (R^{\times}, \cdot, 1)$ and $\mathbb{C}^{\times} = (C^{\times}, \cdot, 1)$ where $R^{\times}$ is the set of all real numbers without 0 and $C^{\times}$ is the set of all complex nubmers without 0.

3. <u>Lanuguare of Ring Theory</u>, also briefly "**language of rings**".
$\mathcal{L} = \{\dot{+}, \dot{-}, \dot{\times}, \dot{0}, \dot{1}\}$.

- Constant symbols: $\dot{0}$, $\dot{1}$.
- Function symbols: Binary function symbols $\dot{+}$, $\dot{-}$, and $\dot{\times}$.

Associated structures:

(a) **The ring of integers** $\mathbb{Z} = (Z, +, -, \cdot, 0, 1)$. The domain of $\mathbb{Z}$ is

$$Z = \text{the set of all integers}$$

Interpretations of $\mathcal{L}$-symbols:
- $\dot{0}^{\mathbb{Z}} = 0 =$ number "zero".
- $\dot{1}^{\mathbb{Z}} = 1-$ number "one"
- $\dot{+}^{\mathbb{Z}} =$ the usual addition of integers, i.e. $\dot{+}^{\mathbb{Z}}(m, n) = m + n$.
- $\dot{-}^{\mathbb{Z}} =$ the usual subtraction of integers, i.e. $\dot{-}^{\mathbb{Z}}(m, n) = m - n$.
- $\dot{\times}^{\mathbb{Z}} =$ the usual multiplication of integers, i.e. $\dot{\times}^{\mathbb{Z}}(m, n) = m \cdot n$.

Similarly we define interpretations $\mathbb{Q} = (Q, +, -, \cdot, 0, 1)$, $\mathbb{R} = (R, +, -, \cdot, 0, 1)$ and $\mathbb{C} = (C, +, -, \cdot, 0, 1)$ where Q, R and C are sets of all rational, real, and complex numbers, respectively.

4. <u>Language of Graphs</u>. $\mathcal{L} = \{\dot{E}\}$ where $\dot{E}$ is a binary relational symbol.

Associated structures:

(a) **Unoriented graphs.** $G = (V, E)$ where V is a set of objects callled **vertices**, and $E \subseteq V \times V$ is a binary relation called the set of **edges**. If $x, y \in V$ we say that x, y are **connected with an edge** iff $(x, y) \in E$ where recall that (x, y) is an ordered pair. The term "unoriented graph" means that if two vertices are connected with an edge we do not distinguish which of them comes the first, that is, be no order should be mentioned. This is achieved by putting both ordered pairs (x, y) and (y, x) into E, and this corresponds to the intuitive understanding that "if x is connected with y then y is connected with x".

A graph $G = (V, E)$ as an $\mathcal{L}$-structure: The domain of the structure is

$$V = \text{ the set of all vertices.}$$

Interpretations of $\mathcal{L}$-symbols:
- $\dot{E}^G = E$, that is $\dot{E}^G$ is the set of all edges.