

## MATH 230

- Review of basic definitions related to groups; the Isomorphism Theorems
- Cosets and Quotient groups
- Actions and orbits, orbit-stabilizer formula
- Euclidean Division Algorithm, Cyclic groups, their subgroups and quotient groups
- Examples of groups:  $D_n$ ,  $S_n$ ,  $A_n$ ,  $GL_n(F)$ ,  $SL_n(F)$  and  $Q_8$
- Finitely generated abelian groups. Classification theorem without proof
- Semidirect products
- Nilpotent and solvable groups, composition series
- Rings, homomorphisms, ideals, quotient rings
- Factorization in commutative rings, primes and irreducibles
- Euclidean domains, PID and UFD
- Fields of quotients
- Rings of polynomials, irreducibility criteria
- Gauss Lemma
- If  $R$  is a UFD then  $R[x]$  is a UFD
- Chinese Remainder Theorem
- Modules, module homomorphisms, quotient modules
- Free modules, rank
- Modules over PID: classification theorem with applications
- Field extensions, degree of an extension, multiplicative property of degrees
- Separable polynomials and splitting fields; algebraic closure
- Cyclotomic extensions
- Finite fields, existence and uniqueness
- The multiplicative group of a finite field is cyclic
- The Fundamental Theorem of Galois Theory
- Cyclotomic and abelian extensions over  $\mathbb{Q}$
- Galois groups of polynomials
- Solvable and radical extensions
- Computation of Galois groups over  $\mathbb{Q}$
- Dual modules
- Tensor products of modules
- Linear representations of groups and modules over the group algebra
- Wedderburn's Theorem and applications to representation theory
- Character theory and orthogonality relations