1. Let $<^*$ be a binary relation defined on the class $[\text{On}]^{<\omega}$ as follows:

$$a <^* b \iff (\exists \alpha \in b - a)[a - (\alpha + 1) = b - (\alpha + 1)].$$

Prove:

(a) $<^*$ is a strict linear ordering on $[\text{On}]^{<\omega}$.

(b) $<^*$ is a well-ordering on $[\text{On}]^{<\omega}$.

(c) If $a \in [\text{On}]^{<\omega}$ then $\{x \in [\text{On}]^{<\omega} \mid x <^* a\}$ is a set.

**Hint.** Concerning (b) and (c), first consider the largest elements of sets $a \in [\text{On}]^{<\omega}$.

2. Let $(Q, \preceq)$ be a partial ordering in the non-strict sense. We say that elements $q, q'$ of $Q$ are comparable if and only if $q \preceq q'$ or $q' \preceq q$. Let $A \subseteq Q$ be a set of pairwise incomparable elements. Show that there is a set $B$ such that $A \subseteq B \subseteq Q$ such that:

(i) $B$ is a set of pairwise incomparable elements;

(ii) Any $q \in Q$ is comparable with some element of $B$.

**Hint.** Use Zorn’s Lemma.

3. Show that the following sets are equinumerous.

(a) $A =$ the set of all infinite sequences $f : \omega \to \omega$. 
(b) \( B = \) the set of all injective infinite sequences \( f : \omega \to \omega \).

(c) \( C = \) the set of all infinite strictly increasing sequences \( f : \omega \to \omega \). (I.e.: 
\( f \) satisfies the condition \( m < n \implies f(m) < f(n) \) for all \( m, n \in \omega \).)

(d) \( D = \) the set of all infinite sequences \( f : \omega \to \{0, 1\} \).

**Hint.** Use the Schröder-Bernstein theorem whenever convenient.