# MATH 280A FALL 2018 HOMEWORK 1

**Target date: Thursday, October 18**

**Rules:** Write as efficiently as possible. Include all relevant points, but do not write too much. Think carefully what to write and what not in order to make the presentation of your argument clear and reasonable. Use common sense to determine the amount of details that need to be included, and keep in mind that your text should correspond to graduate level.

Quote any result from the lecture that comes up in your argument: Do not re-prove these results. If the statement of a problem indicates the maximum allowed length, this length is much more than needed. If you type, do not use font smaller than 10pt.

**I will not grade any text that exceeds the specified length.**

In problems 1-4, try to determine which uses of axioms of set theory are in some sense "essential", point at these uses explicitly, and explain which axioms are used and how. Do not give details on other uses of axioms.

**1. (5 lines)** Show that the intersection of a class with a set is a set.

**2. (7 lines)** Assume $a$ is a set and $f : a \to \mathbf{V}$ is a class function. Show that $f$ is actually a set.

**3. (1/2 page)** Assume $A \subseteq \mathbf{V} \times \mathbf{V}$ is a class. For each $a \in \mathbf{V}$ denote

$$A_a = \{y \mid \langle a, y\rangle \in A\}$$

(i) Show that each $A_a$ is a class.
(ii) Show that the intersection of all classes $A_a$ as well as their union are classes.
(iii) Show that the collection of all $a$ for which $A_a$ is nonempty is a class.

**4. (1/2 page)** Let $R$ be a binary relation which is set-like, and let $A$ be a set. Prove that $\mathsf{pred}_R(A)$ is a set.

**5. (1/2 page)** Let $a$ be a set. Prove the following.

(i) If $a$ is transitive then $\bigcup a$ is transitive.
(ii) $a$ is transitive iff $\mathcal{P}(a)$ is transitive.

**6. (2/3 page)** Prove, without using Axiom of Foundation, that for every $x \in \omega$ we have $x \notin x$. You may use the fact proved in lecture that all elements of $\omega$ are transitive.