

LOGICAL AXIOMS

(TAUT)

ALL TAUTOLOGIES

EQUALITY AXIOMS

(E1)

$t = t$ for any term t

(E2)

$t = t' \longrightarrow (\varphi(x/t) \longleftrightarrow \varphi(x/t'))$ φ atomic

QUANTIFIER AXIOMS

(Q)

$\varphi(x/t) \longrightarrow (\exists x)\varphi$ t substitutable for x in φ

RULES OF INFERENCE

MODUS PONENS

(MP)

$$\frac{\varphi \quad \varphi \longrightarrow \psi}{\psi}$$

QUANTIFIER RULE

(QR)

$$\frac{\varphi \longrightarrow \psi}{(\exists x)\varphi \longrightarrow \psi} \quad x \text{ not free in } \psi$$

THE SET OF AXIOMS A

$$(S1) \quad \neg \mathbf{S}(x) = \mathbf{0}$$

$$(S2) \quad \mathbf{S}(x) = \mathbf{S}(y) \longrightarrow x = y$$

$$(L1) \quad x < \mathbf{S}(y) \longleftrightarrow x < y \vee x = y$$

$$(L2) \quad \neg x < \mathbf{0}$$

$$(L3) \quad x < y \vee x = y \vee y < x$$

$$(A1) \quad x + \mathbf{0} = x$$

$$(A2) \quad x + \mathbf{S}(y) = \mathbf{S}(x + y)$$

$$(M1) \quad x \cdot \mathbf{0} = \mathbf{0}$$

$$(M2) \quad x \cdot \mathbf{S}(y) = (x \cdot y) + x$$