WEEK 1

1. Recall that $\langle u, v\rangle = \{\{u\}, \{u, v\}\}$. Prove that

$$\langle u, v\rangle = \langle u', v'\rangle \quad \text{iff} \quad (u = u' \;\&\; v = v')$$

for all sets u, u', v, v'.

2. Let R be a class relation.

(a) Prove that R is a set if and only if both $\mathrm{dom}(R)$ and $\mathrm{rng}(R)$ are sets.

(b) Assume R is a proper class and A is a set. Is $R[A]$ a set? If so, prove it. If not, give a couterexample.

(c) Assume R is a proper class and A is also a proper class. Is $R[A]$ a proper class? Again, in the affirmative case prove this, otherwise give a couterexample.

(d) Prove that the membership relation is a proper class. More precisely, prove that the class

$$\{\langle x, y\rangle \mid x \in y\}$$

is proper.

3. Let A be a class all of whose elements are transitive sets. Prove that both

$$\bigcup A \quad \text{and} \quad \bigcap A$$

are transitive.

4. Prove the following assertions.

(a) If a is a transitive set then $\bigcup a$ is a transitive set as well.

(b) If a is a transitive set then $\mathcal{P}(a)$ is a transitive set as well.

In either case determine whether the converse holds. In the affirmative case prove the converse, otherwise give a counterexample.

5. Given any set a, prove the following assertions.

e) If every element of a is a transitive set, then $\in\restriction (a \times a)$ is a transitive relation. If a is a transitive set, then also the converse holds.

f) a is transitive iff $a \subset \mathcal{P}(a)$.

g) a is transitive iff $\bigcup a \subset a$.