MATH 281A FALL 2019 HOMEWORK 1

Target date: Wednesday, October 23

Rules: Write as efficiently as possible. Include all relevant points, but do not write too much. Think carefully what to write and what not in order to make the presentation of your argument clear and reasonable. Use common sense to determine the amount of details that need to be included, and keep in mind that your text should correspond to graduate level.

Quote any result from the lecture that comes up in your argument: Do not re-prove these results. If the statement of a problem indicates the maximum allowed length, this length is much more than needed. If you type, do not use font smaller than 10pt.

I will not grade any text that exceeds the specified length.

1. (1/2 page) Work in ZFC. Let $h : \mathbb{B} \to \mathbb{C}$ be a homomorphism of Boolean algebras. Prove that the map $h^* : S(\mathbb{C}) \to S(\mathbb{B})$ between the corresponding Stone spaces defined by

$$h(U) = h^{-1}[U]$$

is continuous, and if h is surjective then h^* is injective.

2. (2/3 page) Work in ZFC. Recall that for a regular cardinal κ, by I_κ we denote the ideal of all bounded subsets of κ.

Assume $(a_\alpha \mid \alpha < \kappa)$ is a sequence of I_κ-positive subsets of κ which are pairwise disjoint modulo I_κ. Use a diagonal argument to construct a set $a \in I_\kappa^+$ which is almost disjoint modulo I_κ with every a_α.

Use the above result to construct, but recursion up to κ^+ an antichain modulo I_κ in the Boolean algebra $\mathbb{B}(\kappa)$.

3. (1/2 page) Work in ZFC. Determine $\mathrm{Sat}(I_\omega)$.

4. (1/2 page) Work in ZFC. Let $\mathbb{B}$ be a Boolean algebra. Assume:

(a) $A \subseteq \mathbb{B}$,
(b) $A{\downarrow} = \{x \in \mathbb{B} \mid x \leq_{\mathbb{B}} a \text{ for some } a \in A\}$ is the downward closure of A in $\mathbb{B}$, and finally
(c) $A^* \subseteq A{\downarrow}$ is an antichain maximal, with respect to the inclusion, among all antichains in $\mathbb{B}$ contained in $A{\downarrow}$.

Prove that if $\bigvee A^*$ exists then

$$\bigvee A^* = \bigvee A{\downarrow} = \bigvee A.$$

5. (1/3 page) Work in ZF. Let κ be a regular uncountable cardinal and let $(A_\xi \mid \xi < \kappa)$ and $(A'_\xi \mid \xi < \kappa)$ be two sequences of subsets of κ such that

$$\{A_\xi \mid \xi < \kappa\} = \{A'_\xi \mid \xi < \kappa\}$$

Prove that

$$\left(\mathop{\Delta}_{\xi<\kappa} A_\xi\right) \triangle \left(\mathop{\Delta}_{\xi<\kappa} A'_\xi\right) \in \mathsf{NS}_\kappa$$

and

$$\left(\mathop{\nabla}_{\xi<\kappa} A_\xi\right) \triangle \left(\mathop{\nabla}_{\xi<\kappa} A'_\xi\right) \in \mathsf{NS}_\kappa$$