1. (1/2 page) Let $\kappa$ be a regular cardinal and $F$ be a uniform normal filter on $\kappa$.
   (a) Prove that $F$ is $\kappa$-complete.
   (b) Prove that $\text{CLUB}_\kappa \subseteq F$, that is, $\text{CLUB}_\kappa$ is the smallest uniform normal ideal on $\kappa$.

2. (1/2 page) Let $\kappa$ be a regular cardinal and $S \subseteq \kappa$. Recall that an ordinal $\alpha$ is a limit point of $S$ iff $\sup(S \cap \alpha) = \alpha$. Also recall that if $S$ is stationary then an ordinal $\alpha < \kappa$ is a reflection point of $S$ iff $S \cap \alpha$ is stationary in $\alpha$. Thus, every reflection point of $S$ is a limit point of $S$ of uncountable cofinality. Define
   \[ \text{succ}(S) = \{ \alpha \in S \mid \alpha \text{ is not a limit point of } S \} \]
   and
   \[ \text{N}(S) = \{ \alpha \in S \mid \alpha \text{ is not a reflection point of } S \} \]
   Prove that if $S$ is a stationary subset of $\kappa$ then $\text{succ}(S) \in \text{NS}_\kappa$ and $\text{N}(S)$ is a stationary subset of $\kappa$.

3. (1/2 page) Recall that we consider ordinals as topological spaces with the interval topology, and similarly the structure $\mathbb{R}$ of all real numbers also as a topological space with the interval topology. Prove the following
   (a) Every continuous map $f : \omega_1 + 1 \to \mathbb{R}$ is eventually constant.
   (b) Every continuous map $f : \omega_1 \to \mathbb{R}$ is eventually constant.
   Clause (a) follows easily, but I am including it for a comparison with clause (b).

4. (1/3 page) Consider a universe where $\omega_1$ is an everyday reality. Assume there is a train network with $\omega_1 + 1$ many stations which are indexed by ordinals below $\omega_1 + 1$, say $(s_\alpha \mid \alpha < \omega_1 + 1)$ is a list of stations such that the trains depart from station $s_0$ and stop at each $s_\alpha$ in the increasing order. The last station is $s_{\omega_1}$.
   Now consider the following situation. At each station $s_\alpha$ one passenger gets off, if there is any on the train, and $\omega$ many get on. It is understood that the passenger who gets off does not reenter.
   Question: How many passengers arrive at station $s_{\omega_1}$?
5. (1/2 page) Let $\alpha, \beta$ be ordinals of uncountable cofinality and $f : \alpha \to \beta$ be a normal cofinal map. Prove the following.

(a) If $S \subseteq \alpha$ is a stationary subset of $\alpha$ then $f[S]$ is a stationary subset of $\beta$.
(b) If $T \subseteq \beta$ is a stationary subset of $\beta$ then $f^{-1}[T]$ is a stationary subset of $\alpha$. 