1. (1/2 page) Let $\mu < \kappa$ be regular cardinals. Define
$$E^\kappa_\mu = \{ \alpha < \kappa \mid \text{cf}(\alpha) = \mu \}.$$ 
(a) Prove that if $\lambda$ is a regular cardinal such that $\mu < \lambda < \kappa$ then the set of all reflection points of $E^\kappa_\mu$ is unbounded in $\kappa$ and closed under limits of cofinality $\lambda$.
(b) Assume $\kappa = \mu^+$. How big is the set of all reflection points of $E^\kappa_\mu$?

2. (1/2 page) Let $\kappa$ be a regular cardinal, $A$ be a set of cardinality larger than $\kappa$ and $C$ be a club in $[A]^{<\kappa}$ in the sense of Jech. Let $\theta$ be a cardinal such that $C \in H_\theta$ and $x \in \mathcal{P}_\kappa(H_\theta)$ be an elementary substructure of $H_\theta$ such that $C \in x$. Prove that $x \cap A \subseteq C$.

3. (2/3 page) Let $\mu < \kappa$ be cardinals and $\kappa$ be regular. Assume $T$ is a tree of height $\kappa$ such that each level of $T$ has cardinality $< \mu$. Prove that $T$ has a cofinal branch.

4. (1/2 page) Let $\kappa$ be regular and $T$ be a tree of height $\kappa$.
   (a) Assume there is a split above every node of $T$. Prove that if $T$ has a cofinal branch than $T$ has an antichain of cardinality $\kappa$.
   (b) Prove that if $T$ is Aronszajn and pruned then there is a split above every node of $T$.
   (c) Assume $\kappa$ is a successor cardinal and $T$ is special. Prove that $T$ is not a Suslin tree.

5. (1/2 page) Let $(L, <)$ be a Suslin line. Recall that we defined an equivalence relation $\sim$ on $L$ as follows:
   $$x \sim y \iff \text{the open interval } (x, y) \text{ is separable}.$$ 
Prove that each equivalence class $[x]$ is a separable interval in $(L, <)$. A singleton is considered an interval here.
6. (1 page) Let $\kappa$ be a cardinal.
   (a) Prove that if $\diamondsuit_\kappa$ holds then $\kappa$ is regular.
   (b) Prove that $\diamondsuit_\kappa$ implies the equality $2^{<\kappa} = \kappa$, so in particular if $\kappa = \mu^+$ we have $2^\mu = \kappa = \mu^+$ and $\diamondsuit \Rightarrow \text{CH}$.
   (c) If $\diamondsuit_\kappa(S)$ holds then there is a family $\mathcal{F}$ of stationary subsets of $S$ such that
      (i) $\text{card}(\mathcal{F}) = 2^\kappa$, and
      (ii) If $S_1, S_2 \in \mathcal{F}$ are such that $S_1 \neq S_2$ then $S_1 \cap S_2$ is bounded in $\kappa$.
      That is, $\text{NS}_\kappa \upharpoonright S$ is as non-saturated as it can possibly be.
   (d) Prove that if the definition of a $\diamondsuit_\kappa^*$-sequence $(A_\alpha \mid \alpha < \kappa)$ is strengthened in that the requirement
      $- \text{card}(A_\alpha) \leq \text{card}(\alpha)$
      is strengthened to
      $- \text{card}(A_\alpha) = 1$
      then this strengthened variant of the $\diamondsuit_\kappa^*$-principle is inconsistent. Prove that even the weaker strengthening where we request
      $- \text{card}(A_\alpha) < \mu$
      for some fixed $\mu < \kappa$ is inconsistent.