WEEK 9

1. Prove that the following two statements are equivalent:

(a) There is a Kurepa tree.

(b) There is a Kurepa family.

Hint. (a) ⇒ (b) Look at the family of all cofinal branches of a Kurepa tree. (b) ⇒ (a) Look at the characteristic functions of the members of a Kurepa family.

2. Let $\kappa$ be regular and let $\mathcal{S} \subseteq \mathcal{P}(\kappa)$ be defined as follows:

$$A \in \mathcal{S} \iff \diamondsuit_{\kappa}(A) \text{ fails.}$$

Show that $\mathcal{S}$ is a uniform ideal on $\kappa$ that is $\kappa$-complete and normal. Notice that $\mathcal{S}$ extends the nonstationary ideal on $\kappa$. (Uniform means that it contains all bounded subsets of $\kappa$.)

Hint. For the proof of the $\kappa$-completeness, resp. the normality of $\mathcal{S}$ use some bijection between $\kappa$ and $\kappa \times \gamma$ for $\gamma < \kappa$, resp. one between $\kappa$ and $\kappa \times \kappa$ as a device that codes families of subsets of $\kappa$ of size $\gamma$, resp. $\kappa$ into subsets of $\kappa$. You can also appeal to Problem 1 from the Week 4 assignment and prove only the normality of $\mathcal{S}$. 