

MATH 282B WINTER 2018 HOMEWORK 2

Target date: Friday, February 9

Rules: Write as efficiently as possible: Include all relevant points and think carefully what to write and what not. Use common sense to determine what is the appropriate amount of details for a course at this level. Quote any result from the lecture you are referring to; do not reprove the result. If the problem indicates maximum allowed length; this is usually much more than needed. If you type, do not use font smaller than 10pt.

I will not grade any text that exceeds the specified length.

Let me point out a subtlety in the definition of κ -stability which I did not stress when introducing the notion; I think I was a bit sloppy with the definition in the lecture.

That an \mathcal{L} -theory T is κ -stable means that for every set of constants \dot{A} of cardinality $\leq \kappa$, and for any complete $\mathcal{L}_{\dot{A}}$ -theory $T_{\dot{A}} \supseteq T$, the cardinality of $S_1(T_{\dot{A}})$ is at most κ .

So we are fixing a complete theory $T_{\dot{A}}$ extending T which is then contained in every type in $S_1(T_{\dot{A}})$. This makes the definition non-trivial, as otherwise we would be considering trivialities like this: For a language \mathcal{L} with a binary relation symbol R we could get, for each $X \subseteq \dot{A}$, a type $p_X(v) \in S_1(T_{\dot{A}})$ by demanding that “ vRa ” is in p_X for every $\dot{a} \in X$ and “ $\neg vRa$ ” is in p_X for all $\dot{a} \in \dot{A} \setminus X$. This would generate continuum many different types for every countable \dot{A} in a trivial way. But if we fix $T_{\dot{A}}$ which makes some restrictions on objects denoted by constants from \dot{A} then many of p_X will contradict the statements of $T_{\dot{A}}$, so they will not be types, and this triviality will not occur.

An equivalent, more semantic way of defining the notion of κ -stability of T , is to demand that for every $\mathcal{M} \models T$ and every $A \subseteq \mathcal{M}$ of cardinality $\leq \kappa$, the cardinality of $S_1^{\mathcal{M}}(A)$ is at most κ .

- 1. (1/2 page)** Prove that DLO is not κ -stable for any infinite cardinal κ . This can be done directly, or “from scratch”, so please **do not** follow the suggestion from Exercise 5.2.1 in the book to use Exercise 8.2.8, but do a direct argument.
- 2. (1 page)** Book, Exercise 5.2.2
- 3. (2/3 page)** Book, Exercise 5.2.3. Let me stress: it is crucial that the language L is at most countable.
- 4. (1/2 page)** Book, Exercise 5.2.4