MATH 282B WINTER 2018 HOMEWORK 4

Target date: March 26, 2018

Rules: Write as efficiently as possible: Include all relevant points and think carefully what to write and what not. Use common sense to determine what is the appropriate amount of details for a course at this level. Quote any result from the lecture you are referring to; do not reprove the result. If the problem indicates maximum allowed length; this is usually much more than needed. If you type, do not use font smaller than 10pt.

I will not grade any text that exceeds the specified length.

1. (1/3 p) Let $\mathcal{L}$ be the language of arithmetic as in Homework 3, Problem 2. In Homework 3, Problem 4 you proved that if $\mathcal{N}$ is the standard model of arithmetic and $\mathcal{N}'$ is an elementary extension of $\mathcal{N}$ then the pair $(\mathcal{N}', \mathcal{N})$ is not a Vaughtian pair.

Now prove that the theory $\mathsf{Th}(\mathcal{N})$ does have a Vaughtian pair.

2. (1/2 p) Book, Exercise 5.5.6. To simplify the notation, put $n = 1$, that is, work just with formula $\varphi(x, \bar{y})$ instead of $\varphi(x_1, \ldots, x_n, \bar{y})$.

3. (2/3 p) Book, Exercise 5.7.1.

4. (1 p) Book, Exercise 5.7.2.

5. (2/3 p) Book, Exercise 6.1.13 Part 1.

6. (1/2 p) Book, Exercise 6.1.14 Part 1. Rational functions are functions of the form

$$\frac{p(x_1, \ldots, x_\ell)}{q(x_1, \ldots, x_\ell)}$$

where p, q are polynomials.

7. (2/3 p) Book, Exercise 6.2.1.