1. If \( U \) is a normal measure on \( \kappa \) w.r.t./over \( M = \mathbb{ZFC}^- \), say \( M = J_\kappa \), then \( \kappa \) is well-founded part of \( \text{ult}(M, U) \). We are not assuming \( U \in M \).

2. \( U \) is weakly amenable w.r.t. \( M \) iff
   a) If \( f: \kappa \to \mathcal{P}(\kappa) \) and \( f \in M \)
      then \( \exists \xi < \kappa \mid f(\xi) \cap U \in M \).
   b) \( \mathcal{P}(\kappa) \cap M = \mathcal{P}(\kappa) \cap \text{ult}(M, U) \).

3. \( U \) is weakly amenable w.r.t. \( M \) then \( \kappa \) is weakly compact in \( M \), and
   \( \exists \alpha < \kappa \mid \alpha \text{ weakly compact in } M_\alpha \cap U \).

4. \( M, M' \) transitive, arbitrary.
   \( \sigma: M \to M' \) \( \Sigma_0 \)-preserving and cofinal
   a) \( \sigma \) is \( \Sigma_1 \)-preserving
   b) Let \( A \in M \) s.t. \( (M, A) \) is amenable, i.e.
      \( x \cap A \in M \) all \( x \in M \).
      Let \( A' = \bigcup \{ \sigma(A \setminus x) \mid x \in M \} \).
      Then \( (M', A') \) is amenable and
      \( \sigma: (M, A) \to (M', A') \) is \( \Sigma_0 \)-preserving
      w.r.t. language \( \exists \in A' \).
5. Let $(M, A)$ be amenable, $U$ be an ultrafilter over $M$, and $\text{Ult}((M, A), U)$ well-founded, say $(M', A') = \text{Ult}((M, A), U)$. Then $A'$ is as in (4).

6. If $M = \langle J_c, U \rangle$ is a premouse and $\langle M_i : i < \lambda \rangle$ is an iteration then the ultrapower maps are
   - fully elementary in the language $\mathcal{L}$.
   - $\Sigma_0$-preserving in the language $\mathcal{L}$, $U$.
   - cofinal, hence
     $\Sigma_1$-preserving in the language $\mathcal{L}$, $U$.

7. "Shift" Lemma:
   (Successor step in the copying construction)
   Let $\sigma: \overline{M} \to M$ be $\Sigma_0$-preserving, $\sigma(\overline{\kappa}) = \kappa$, $\overline{U}$ a normal measure over $\overline{M}$ on $\overline{\kappa}$, $U$ a normal measure over $M$ on $\kappa$. s.t. $x \in \overline{U} \implies \sigma(x) \in U$. Let
   $M' = \text{Ult}(M, U)$ and $\overline{M}' = \text{Ult}(\overline{M}, \overline{U})$. 
Then there is a unique \( \Sigma_0 \)-preserving map \( \sigma' : \overline{M} \to M' \) s.t.

- \( \sigma'(\overline{\kappa}) = \kappa \)
- The diagram

\[
\begin{array}{ccc}
M & \xrightarrow{\pi} & M' \\
\uparrow \sigma & & \uparrow \sigma' \\
\overline{M} & \xrightarrow{\pi} & \overline{M}'
\end{array}
\]

commutes.

The map is defined by

\[
\sigma'(\pi(f)(\overline{\kappa})) = \pi \circ \sigma(f)(\kappa)
\]

In fact:

\[
\sigma '\upharpoonright \overline{\kappa} + M = \sigma \upharpoonright \overline{\kappa} + M
\]

8. Use the shift lemma to complete the proof of the copying construction.

Find an example of \( J_\kappa \) and a normal measure (in ZFC) \( U \) over \( J_\kappa \) s.t. \( U \) on \( \kappa \), \( J_\kappa \models \text{ZFC}^- + \kappa^+ \) exists such that \( J_\kappa \) can see \( \mathcal{P}(\kappa) \) as an element and s.t. \( \text{Ult}(J_\kappa, U) \) well-founded, but \( U \) not weakly amenable over \( J_\kappa \).
9. If $\mathcal{T} = (\kappa^+)^J_{\alpha^*}$ then $J_{\mathcal{T}} \models \text{ZFC}$.

10. If $\mathcal{T} = (\kappa^+)^J_{\alpha^*}$ and $f: \kappa \to \gamma$ is a surjection then there is $a \in \kappa$ that codes a well-ordering of order-type $\gamma$.

Easier: $a \in \kappa \times \kappa$ s.t. $a$ is a well-ordering of order-type $\gamma$.

11. If $\mathcal{T} = (\kappa^+)^J_{\alpha^*}$, $a \in J_{\alpha^*}$ and $a \in \kappa$ then $a \in J_{\mathcal{T}}$ (like the proof of $\text{GCH}$).

12. $p \in R_{\alpha^*} \Rightarrow p \in P_{\alpha^*}$

(diagonalization argument.)

13. $P_{\alpha^*}$ is a $\Sigma_1$-cardinal over $J_{\alpha^*}$.

14. If $\kappa$ is a cardinal in $J_{\alpha^*}$ then $(H_\kappa)^{J_{\alpha^*}} = J_{\kappa}$.

15. $<^* \text{ is a well-ordering on } [\Omega]^\omega_{\omega}$.

16. Try to check the details about $B_{\mathcal{T}}$. 
17. Prove $B_t$ is closed and 
    $\bar{e} \in B_t \Rightarrow B_{\bar{e}} = B_t \cap \bar{e}$. 

18. Try to prove the claim: 
    $t^* \leq \bar{e}$ in $B_t \Rightarrow S_{t^*} < S_{\bar{e}}$. 