Exercise 1.3

1. Let $E$ be a $(n, n)$ matrix. Recall the object $	ext{rank}(E) = (f_1, f_2, \ldots, f_k)$ where, letting $
u = (\nu_1, \nu_2, \ldots, \nu_k)$ be a non-zero vector, we have

$$T: \text{Vec}(\nu) \to \text{Vec}(\nu)$$

where $T(\nu) = E \nu$ and $1 < \nu_1 < \nu_2 < \ldots < \nu_k$.

Proof: Let $E = \begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{nn} \end{pmatrix}$ be a non-zero matrix.

(a) Prove that $E$ is invertible.

(b) Prove that $E$ is not invertible.

(c) Prove that $E$ is singular.

(d) Prove that $E$ is nonsingular.

2. Let $i: M \to N$ be an elementary embedding and $E = \{e_1, e_2, \ldots, e_k\}$ be a subset of $M$.

Prove: If $E$ is a proper subset of $M$, then $E$ is not an elementary embedding of $M$ into $N$.

3. Prove that if $E$ is an elementary embedding of $M$ into $N$, then $E$ is injective.

4. Let $M$ be a transitive set. Recall the object $\text{rank}(M) = (f_1, f_2, \ldots, f_k)$ where, letting $\nu = (\nu_1, \nu_2, \ldots, \nu_k)$ be a non-zero vector, we have

$$T: \text{Vec}(\nu) \to \text{Vec}(\nu)$$

where $T(\nu) = E \nu$ and $1 < \nu_1 < \nu_2 < \ldots < \nu_k$.

Proof: Let $E = \begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{nn} \end{pmatrix}$ be a non-zero matrix.

(a) Prove that $E$ is invertible.

(b) Prove that $E$ is not invertible.

(c) Prove that $E$ is singular.

(d) Prove that $E$ is nonsingular.
Proof:

Let $E$ be a $(\mathcal{X}, \mathcal{Y})$-extension of $\mathbb{N}$ extended M. As usual, $\mathbb{N}$ is well-founded, and $\mathcal{X}$ is a function such that $\mathcal{X}(\mathbb{N}) = \mathbb{N}$. Therefore, $\mathbb{N}$ is well-founded.

Every element of $\mathbb{N}$ is $\mathcal{Y}$-well-founded in $\mathbb{N}$.

Exercise 3.7.2012 (a)
But the great wooden conclusion is not made:

A more direct any wooden conclusion in thinking:

\[ N \geq 9 \]

What if \( f(t) \) \( (t) < 0 \) \( ? \)

\[ n = \frac{1}{2} + \frac{1}{2} \]

\[ \forall (E) \]

And can selection \( E \epsilon V \) set.

\[ A = \{ \text{wooden} \} \]

\( N \in \mathbb{N} \)

\( \forall \in \mathbb{N} \)

\( \forall (E) \in \mathbb{N} \)

Proof that the other line of 0 being

Exercises 13