Southern California Number Theory Day: **Abstracts**
UC Irvine, September 24, 2022

Aaron Landesman, Harvard University

Arithmetic Representations on Generic Curves

Over the last century, the Hodge and Tate conjectures have inspired much activity in algebraic and arithmetic geometry. These conjectures give predictions for when certain topological objects come from geometry. Simpson and Fontaine-Mazur introduced non-abelian analogs of these conjectures. We prove these analogs for low rank local systems on generic curves, resolving conjectures of Esnault-Kerz and Budur-Wang as well as answering questions of Kisin and Whang.

Michelle Manes, University of Hawaii

Mahler Measure and Arithmetic Dynamics

The Mahler measure of a polynomial can be viewed as a kind of height function, and for polynomials with integer coefficients it is directly related to the (Weil) height of the roots. Mahler measure of multivariate polynomials appears to have deep and surprising ties to the geometry of the corresponding varieties. More recently, a dynamical Mahler measure (relative to a polynomial f) was defined that relates to the f-canonical height of points rather than the Weil height. My collaborators and I have defined a multivariate dynamical Mahler measure and proven several results that maintain the flavor of classical results in the theory of Mahler measure.

This talk will give a tour of single- and multi-variable Mahler measure, and then describe some dynamical analogs. It will be heavy on examples, motivation, and analogy and light on technical details.

Holly Swisher, Oregon State University

Generalized Ramanujan-Sato series arising from modular forms

In 1914, Ramanujan gave several fascinating infinite series representations of $1/\pi$. In the 1980's it was determined that these series provide efficient means for approximating π. Since then discovering and proving series of this type have been of interest. Motivated by work of Chan, Chan, and Liu, we obtain a new general theorem yielding corollaries that produce generalized Ramanujan-Sato series for $1/\pi$. We use these corollaries to construct explicit examples arising from modular forms on arithmetic triangle groups. This work is joint with Angelica Babei, Lea Beneish, Manami Roy, Bella Tobin, and Fang-Ting Tu. It was initiated as part of the Women in Numbers 5 workshop.

Stanley Xiao, University of Northern British Columbia

Counting elliptic curves with a rational 2-torsion point ordered by conductor

Recently, A. Shankar, A.N. Shankar, and X. Wang counted certain (conjecturally) large families of elliptic curves when ordered by conductor. Such a feat is possible only with certain severe restrictions. Specifically, they assumed that the j-invariants of the curves E under consideration are $O(\log |\Delta(E)|)$ and that the Szpiro ratio of the curves are smaller than 7/4. In this talk I will discuss my recent work on the analogous problem of counting elliptic curves with a (marked) rational 2-torsion point. We note that

1. we do not need any assumptions on the size of the j-invariant; and
2. we can push the valid range for the Szpiro ratio to $9/4 + \kappa$ for $\kappa > 0$.

The significance of the positivity of κ is that 9/4 is the natural limit of the geometry of numbers method for this family, so obtaining an exponent strictly larger than 9/4 requires non-trivial input from beyond the geometry of numbers.

Time permitting, I will also discuss work on estimating the size of the average rank of the curves in this family with respect to this ordering.

Lightning Talks

Gilyoung Cheong, University of California, Irvine

The distribution of the cokernel of a random p-adic integral matrix

Let p be a prime. Friedman and Washington computed the distribution of the cokernel of a Haar-random $n \times n$ matrix over the p-adic integers. When n goes to infinity, this distribution converges to the Cohen–Lenstra distribution, which conjecturally predicts the distribution of the p-part of a random quadratic imaginary number field. Since then, this was followed by many other interesting results that computed various asymptotic distributions of cokernels of random p-adic integral matrices when n goes to infinity, but there have been less effort in understanding similar behaviors when n is fixed. In this talk, I will present some surprising results when n is fixed with some explicit exact formulas. This talk is based on two separate joint works, one with Nathan Kaplan and another with Yunqi Liang and Michael Strand, both of which extend former joint work with Yifeng Huang.

Rusiru Gambheera, University of California, San Diego

An Unconditional Equivariant Main Conjecture in Iwasawa Theory

In 2015 Greither and Popescu constructed a new class of Iwasawa modules, which are the number field analogues of $p-$adic realizations of Picard 1- motives constructed by Deligne. They proved an equivariant main conjecture by computing the Fitting ideal of these new modules over the appropriate profinite group ring. This is an integral, equivariant refinement of Wiles' classical main conjecture. As a consequence they proved a refinement of the Brumer-Stark conjecture away from 2. All of the above was proved under the assumption that the relevant prime p is odd and that the appropriate classical Iwasawa μ-invariants vanish. Recently, Dasgupta and Kakde proved the Brumer-Stark conjecture, away from 2, unconditionally, using a generalization of Ribet's method. We use the Dasgupta-Kakde results to prove an unconditional equivariant main conjecture, which is a generalization of that of Greither and Popescu. As applications of our main theorem we prove a generalization of the main result of Dasgupta-Kakde and we compute the Fitting ideal of a certain naturally arising Iwasawa module. This is joint work with Cristian Popescu.

Hua Lin, University of California, Irvine

One-level density of zeros of Dirichlet L-functions over function fields

We compute the one-level density of zeros of order $\ell \geq 3$ Dirichlet L-functions over function fields $\mathbb{F}_q[t]$ for $\ell = 3, 4$ in the Kummer setting ($q \equiv 1 \pmod{\ell}$) and for $\ell = 3, 4, 6$ in the non-Kummer setting ($q \not\equiv 1 \pmod{\ell}$). In each case, we obtain a main term predicted by Random Matrix Theory (RMT) and a lower order term not predicted by RMT. We also confirm the symmetry type of the family is unitary, supporting the Katz and Sarnak philosophy.