Southern California Number Theory Day: **Abstracts**

UC Irvine, October 7, 2023

Carl Wang-Erickson, University of Pittsburgh

Moduli stacks of Galois representations

There has recently been a wide variety of development and applications of moduli stacks of Galois representations. The goal of this talk is to introduce the salient features of these moduli stacks as concretely as possible, focusing on the example of 2-dimensional representations, continuous with respect to a p-adic topology, of the absolute Galois group of $\mathbf{Q}_p$. In this setting, I will describe a geometrization of the p-adic local Langlands correspondence for $\mathrm{GL}_2/\mathbf{Q}_p$ that is joint work with Christian Johannson and James Newton.

Nadia Heninger, UCSD

Some recent cryptographic applications of number theory

I will survey some fun applications of number theory that have arisen in my recent work in cryptography.

Kyle Pratt, BYU

Rational points on Erdős-Selfridge curves

Many problems in number theory are equivalent to determining all of the rational points on some curve or family of curves. In general, finding all the rational points on any given curve is a challenging (even unsolved!) problem.

The focus of this talk is rational points on so-called Erdős-Selfridge curves. A deep conjecture of Sander, still unproven in many cases, predicts all of the rational points on these curves.

I will describe work-in-progress proving new cases of Sander's conjecture, and sketch some ideas in the proof. The core of the proof is a 'mass increment argument,' which is loosely inspired by various increment arguments in additive combinatorics. The main ingredients are a mixture of combinatorial ideas and quantitative estimates in Diophantine geometry.

Francesc Castella, UCSB

On Kolyvagin's conjecture and its refinement

Let $E/\mathbb{Q}$ be an elliptic curve, and $p > 2$ a prime of good ordinary reduction. In 1991, Kolyvagin conjectured the non-triviality of a system of cohomology classes derived from Heegner points on E of varying conductors. The first major result towards Kolyvagin's conjecture is due to W. Zhang, who obtained a proof of the conjecture under certain ramification hypotheses on $E[p]$. In this talk, I will explain a new proof of Kolyvagin's conjecture building on Iwasawa theoretic techniques and the work of Cornut-Vatsal. Our result treats the cases where $E[p]$ is irreducible as a Galois module (with no ramification hypotheses) as well as the first cases where E admits a rational p-isogeny. Moreover, by the same methods we also prove a refinement of Kolyvagin's conjecture posed by W. Zhang in 2014. Based on a joint work with A. Burungale, G. Grossi, and C. Skinner.

Lightning Talks

Yifeng Huang, University of British Columbia

Zeta functions on orders

(Based on joint work with Ruofan Jiang) The Riemann zeta function generalizes to the Dedekind zeta function of any number field, which is classically known to have a meromorphic continuation and a functional equation. Much more recently known, the Dedekind zeta function generalizes to orders and these properties persist (Yun, 2013). In this talk, I discuss two analogous zeta functions on orders of function fields and their relations to geometry and combinatorics.

Tynan Ochse, Art of Problem Solving

Weil reciprocity for rigid analytic curves

We generalize classical Weil reciprocity and Deligne's "symbole modere" to partially proper compactifiable rigid analytic curves. We define the Robba ring of such curves, which admits a suitably nice lattice theory, and whose group of units serve as a nonarchimedean analogue of the loop group. Together with standard gerbe constructions, this framework allows us to derive a rigid analytic version of the product formula and the duality statement of Weil reciprocity. Future applications of the result include deriving a rigid analytic version of categorical geometric local class field theory.

Harold Polo, University of California, Irvine

Goldbach conjecture for polynomials

We discuss analogues of Goldbach conjecture for many classes of polynomials. In particular, we focus on polynomials with positive integer coefficients. This talk is based on two separate joint works, one with Sophia Liao and another with Nathan Kaplan.