Southern California Number Theory Day: **Abstracts**

UC Irvine, October 18, 2025

Sarah Peluse, Stanford University

Integer Distance Sets

I'll speak about joint work with Rachel Greenfeld and Marina Iliopoulou in which we address some classical questions concerning the size and structure of integer distance sets. A subset of the Euclidean plane is said to be an integer distance set if the distance between any pair of points in the set is an integer. Our main result is that any integer distance set in the plane has all but a very small number of points lying on a single line or circle. From this, we deduce a near-optimal lower bound on the diameter of any non-collinear integer distance set of size n and a strong upper bound on the size of any integer distance set in $[-N, N]^2$ with no three points on a line and no four points on a circle.

Jared Duker Lichtman, Stanford University

The abc conjecture almost always

The celebrated abc conjecture asserts that every solution to the equation $a + b = c$ in triples of coprime integers (a, b, c) must satisfy $\text{rad}(abc) \geq c^{1-\epsilon}$, with finitely many exceptions. We prove a power-saving bound on the exceptional set of such triples. Namely, there are at most $O(X^{33/50})$ many triples of coprime integers in a cube $(a, b, c) \in [1, X]^3$ satisfying $a + b = c$ and $\text{rad}(abc) < c^{1-\epsilon}$. The proof is based on a combination of bounds for the density of integer points on varieties. Joint work with Tim Browning and Joni Teravainen.

Yunqing Tang, University of California, Berkeley

The arithmetic of power series and applications to irrationality

We will discuss a new approach to prove irrationality of certain periods, including the value at 2 of the Dirichlet L-function associated to the primitive quadratic character with conductor -3. Our method uses rational approximations from the literature and we develop a new framework to make use of these approximations. The key ingredient is an arithmetic holonomy theorem built upon earlier work by André, Bost, Charles (and others) on arithmetic algebraization theorems via Arakelov theory. We will also discuss the recent result on irrationality measures. This is joint work with Frank Calegari and Vesselin Dimitrov.

Vesselin Dimitrov, Caltech

The Arithmetic of Power Series and Applications to Diophantine Analysis

This will be the second talk at SCNTD about an ongoing collaboration with Frank Calegari and Yunqing Tang on arithmetic holonomy bounds. We will explain how those bounds get refined from qualitative linear independence statements to quantitative Diophantine inequalities. Applying these on a simple dihedral construction leads directly to a new proof of the transcendency of Pi, but more interestingly in the algebraic situation, to an explicit lower bound on linear forms in two logarithms and rational variables. This bound is sufficiently uniform to derive, via Bombieri's geometry of numbers argument, a new and easy proof of the fundamental effective theorem on the Diophantine approximation by a finitely generated subgroup of the multiplicative group over a number field. The latter includes as special cases an effective height upper bound on the two-variable S-unit equation, the algorithmic resolution of the Thue—Mahler and hyperelliptic equations, and the Baker-Feldman effective sub-Liouville exponent for a general algebraic number of degree at least 3.

Lightning Talks

Bryan Hu, UCSD

Special Values of L-functions for Quaternionic Groups

We discuss algebraicity results for L-functions associated to automorphic forms on quaternionic groups. Quaternionic modular forms (QMFs) are automorphic forms on these groups whose real components lie in the quaternionic discrete series. QMFs have a robust theory of Fourier coefficients developed by Gross-Wallach, Gan-Gross-Savin, and Pollack. We will explain how recent work on arithmetic properties of these Fourier coefficients, and an analog of Maass-Shimura operators for exceptional groups, can be applied to obtain algebraicity results for certain L-functions.

Chris Xu, UCSD

The essence of Diophantine geometry, in one simple diagram

For a variety X over the rational numbers, consider $X(\mathbb{Q})$ sitting inside X. Although $X(\mathbb{Q})$ is currently invisible to us, I have equipped you with an assortment of special "flashlights" to shine on this picture. What each flashlight will reveal is that $X(\mathbb{Q}) \subset X$ is sitting inside a visible inclusion $V \subset U$ of "cohomology varieties", with V and U depending on the flashlight you used. In fact, by combining information from each flashlight, we are expected to determine $X(\mathbb{Q})$ on the nose. Curious? Tune in to this talk to learn more!

Connor Lane, UCSB

AGM Aquariums and Elliptic Curves Over Arbitrary Finite Fields

Let $q = p^n$ with p odd and let $k = \mathbb{F}_q$. The AGM aquarium of k is a graph where the vertices are elements of k^2 and there are edges between (a, b) and $((a+b)/2, \sqrt{ab})$. In 2021, Michael J. Griffin, Ken Ono, Neelam Saikia, and Wei-Lun Tsai noticed a surprising relationship between this setup and elliptic curves in the case where $q = 3 \mod 4$. We generalize their results to arbitrary odd q and discover rich new behavior of the AGM aquarium in this case. This work is joint with June Kayath, Ben Neifeld, Tianyu Ni, and Hui Xue.

Jasmine Camero, Emory University

Classifying Possible Density Degree Sets of Hyperelliptic Curves

Let C be a smooth, projective, geometrically integral hyperelliptic curve of genus $g \geq 2$ over a number field k. To study the distribution of degree d points on C, we introduce the notion of $\mathbb{P}^1$- and AV-parameterized points, which arise from natural geometric constructions. These provide a framework for classifying density degree sets, an important invariant of a curve that records the degrees d for which the set of degree d points on C is Zariski dense. Zariski density has two geometric sources: If C is a degree d cover of $\mathbb{P}^1$ or an elliptic curve E of positive rank, then pulling back rational points on $\mathbb{P}^1$ or E give an infinite family of degree d points on C. Building on this perspective, we give a classification of the possible density degree sets of hyperelliptic curves.

Claire Levaillant, USC

Solutions to the Diophantine equation $\sum_{i=1}^{n} \frac{1}{x_i} = 1$ in integers $p^a q^b$ with p and q two set primes and $a, b \geq 0$

We investigate the solutions to $\sum_{i=1}^{n} \frac{1}{x_i} = 1$ with only two set prime divisors involved in the denominators.

Sehun Jeong, Claremont McKenna College

Primitive elements in number fields and Diophantine avoidance

The famous primitive element theorem states that every number field K is of the form Q(a) for some element a in K, called a primitive element. In fact, it is clear from the proof of this theorem that not only there are infinitely many such primitive elements in K, but in fact most elements in K are primitive. This observation raises the question about finding a primitive element of small "size", where the standard way of measuring size is with the use of a height function. We discuss some conjectures and known results in this direction, as well as some of our recent work on a variation of this problem which includes some additional avoidance conditions. Joint work with Lenny Fukshansky at Claremont McKenna College.

Roberto Hernandez, UCI

Rational Points on a Family of Genus 3 Hyperelliptic Curves

We compute the rational points on certain members of the following family of hyperelliptic curves

$$C_a\colon y^2 = x^8 + (4 - 4a^4)x^6 + (8a^4 + 6)x^4 + (4 - 4a^4)x^2 + 1$$

via the method first developed by Dem'yanenko and then further generalized by Manin. In particular, we show that the method of Chabauty-Colemanm, while applicable to certain members of this family, is not the most effective way of computing $C_a(\mathbb{Q})$. We adapt the approach of Kulesz, incorporating root numbers to further restrict the possible ranks of the elliptic curves arising in the Jacobian decomposition.