Math 206A: Algebra
Midterm 1
Friday, October 30, 2020.

- You have **90 minutes** for this exam. Pace yourself, and do not spend too much time on any one problem.
- **Show your work and justify all of your answers!** The more you explain your thought process, the easier it will be to give partial credit for incomplete solutions.
- This is a closed-book exam. No notes or outside resources can be used.
  Do not use a calculator.
- You may use results that we proved in lecture without proving them here provided you clearly state the result you are using.
  (There is an exception: If a question asks you to prove something that we proved in lecture, you should prove it, don't just state it.)

Problems	
1 (5 Points)	
2 (3 Points)	
3 (5 Points)	
4 (10 Points)	
5 (10 Points)	
Total	

Problems	
6 (5 Points)	
7 (5 Points)	
8 (10 Points)	
9 (10 Points)	
10 (12 Points)	
Total	

Problems

1. State the First Isomorphism Theorem.
2. What is the order of the automorphism group of $\mathbb{Z}/8\mathbb{Z}$?
   No explanation is necessary, you can just write a number.
3. For which integers $n \geq 2$ is the group $\{\mathrm{id}, (12)\}$ a normal subgroup of S_n?
   Prove that your answer is correct.
4. (a) **Either prove that the following statement is true or give a counterexample showing that it is false**: Suppose G is a group. If H is a normal subgroup of G and K is a normal subgroup of H, then K is a normal subgroup of G.
   (b) **Either prove that the following statement is true or give a counterexample showing that it is false**: Suppose G is a group and H, K are subgroups of G such that $K \leq H$. If K is a normal subgroup of G, then K is a normal subgroup of H.
5. Show that for any $n \geq 3$, A_n contains a subgroup isomorphic to S_{n-2}.
6. Let G be a finite group and $g \in G$. Let $\mathcal{K}$ be the conjugacy class of g.
   Show that $|\mathcal{K}|$ divides $|G|$.
7. **Either prove that the following statement is true or give a counterexample showing that it is false**: Suppose that G_1 and G_2 are finite groups such that for each positive integer n, G_1 and G_2 have the same number of conjugacy classes of size n. Then G_1 and G_2 are isomorphic.
8. Let G be a finite nontrivial p-group. Prove that $Z(G)$ is nontrivial.
9. State Sylow's Theorem.
10. (a) Let G be a group and $x \in G$ have order k. Prove that $x^n = 1$ if and only if $k \mid n$.
    (b) Suppose G is a group and $x, y \in G$ satisfy $xy = yx$. Suppose that the order of x is n and the order of y is m where $\gcd(n, m) = 1$. Prove that the order of xy is nm.