Math 206A: Algebra
Homework 4

Due Friday, November 6th at 12PM.
Please email `nckaplan@math.uci.edu` with questions.

The first exercise comes from an old UCI Comprehensive Exam. All other exercises are from Dummit and Foote.

1. Spring 2006 Comprehensive Exam #1.
   Prove that all groups of order 12 are solvable. Are they all abelian?
   (You can use any facts we proved in lecture without proving them again.)

2. Exercises 3, 4, and 5 of Section 4.4.
   These problems ask you to prove some of the claims I made in lecture about the automorphism groups of D_8 and Q_8.

3. Exercise 6,7, and 8 of Section 4.4.
   A subgroup H of a group G is called *characteristic* in G, denoted H char G, if every automorphism of G maps H to itself. That is, $\sigma(H) = H$ for all $\sigma \in \text{Aut}(G)$.

   This definition comes from the top of page 135 of Dummit and Foote. Some main properties of characteristic subgroups are then stated. You prove them in these exercises. The third property is particularly useful (see for example the proof of Proposition 21 on page 145).

4. Exercise 12 of Section 4.5.
   In this exercise you analyze Sylow 2-subgroups of dihedral groups.

5. Exercise 10 of Section 2.4 and Exercises 9, 10, and 11 of Section 4.5.
   In these exercises you consider the Sylow 2-subgroups and Sylow 3-subgroups of the matrix group $\text{SL}_2(\mathbb{Z}/3\mathbb{Z})$ and then you prove that $\text{SL}_2(\mathbb{Z}/3\mathbb{Z})/Z(\text{SL}_2(\mathbb{Z}/3\mathbb{Z})) \cong A_4$.

6. Exercise 16 of Section 4.5.
   In this exercise you prove that there are no simple groups of order pqr where p, q, r are distinct primes.

7. Exercise 22 of Section 4.5.
   This is a classic Algebra Qualifying Exam type problem: 'Show that no group of order (some number) is simple.'