Math 206A: Algebra
Additional Midterm 1 Practice Problems

Here are some additional practice problems to help you prepare for Midterm 1.

1. Exercise 4 Section 3.4:
   Use Cauchy's theorem and induction to show that a finite abelian group G has a subgroup of order n for each positive divisor of $|G|$.

2. Exercises 1,2, and 3 of Section 4.1:
   A group action is called *transitive* if there is exactly one orbit of the action. These three exercises give some basics of transitive group actions. For Exercise 2, Dummit and Foote use G_a to denote the stabilizer of a– I have been avoiding this notation in lecture.

3. Exercise 33 of Section 4.3:
   In this exercise you compute the sizes of the different conjugacy classes in S_n. You will not see anything involving this much counting on the midterm, but this is good practice for getting comfortable working with groups of permutations.

4. Spring 2019 Comprehensive Exam #9b:
   Either prove the following statement or give a counterexample showing that it is false: If n divides the order of a finite group G, then there exists $g \in G$ of order n.

5. Spring 2019 Comprehensive Exam #9d:
   Either prove the following statement or give a counterexample showing that it is false: If the group G acts on the set S, and if $s \in S$ is an element, then the stabilizer of s is a normal subgroup of G.

6. Spring 2014 Comprehensive Exam #1:
   Suppose G is a finite group, $H \leq G$ and $a \in G$ is such that $a^k \in H$ for some integer k with $\gcd(k, |G|) = 1$. Prove that $a \in H$.

7. Spring 2014 Comprehensive Exam #2:
   Describe all permutations in S_n that commute with $(1, 2, \ldots, n)$.

8. Spring 2013 Comprehensive Exam #2:
   Let $n \geq 3$ and let $\sigma \in S_n$ be an $(n-1)$ cycle. Show that if $\tau \in S_n$ commutes with σ then τ is in the cyclic subgroup generated by σ.

9. Spring 2007 Comprehensive Exam #1:
   Show that A_4 violates the converse of Lagrange's theorem.

10. Spring 2005 Comprehensive Exam #G3:
    Is there an example of a group G whose subgroups are all normal, but G is not abelian? If not, prove it. If so, describe one such example.