Math 206B: Algebra
Homework 1

Due Friday, January 15th at 12:00PM.
Please email nckaplan@math.uci.edu with questions.

So far in Math 206B we have discussed Sections 7.5 (Rings of Fractions) and Section 7.6 (Chinese Remainder Theorem). On this homework you will see some problems about these topics, but in order to solve them it is helpful to use some results from earlier in Chapter 7. All exercises are from Dummit and Foote.

1. Exercises 25 and 26 of Section 7.3.
   The first exercise asks you to prove the Binomial Theorem in a general commutative ring with identity. The next exercise introduces the *characteristic* of a ring. I mentioned this last quarter when talking about one of the problems from the Sample Final Exam. A few of the problems later on this homework will use ideas from these two exercises.
   Hint: For Exercise 25, you may want to use the basic property of binomial coefficients $\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$ and argue by induction.

2. Exercises 27 and 28 of Section 7.3.
   In these exercises you compute the characteristic of a Boolean ring and prove an important property about the characteristic of an integral domain.

3. Exercise 14 of Section 7.4.
   In this exercise you write down representatives for the elements of the quotient of a polynomial ring by an ideal generated by a monic polynomial. I put this exercise as one of the 'Suggested Problems' for the Final Exam for 206A, so maybe some of you have solved it already. We will work extensively with quotients of polynomial rings in the weeks ahead.

4. Exercises 3 and 4 of Section 7.5.
   These exercises allow you to use what you know about rings of fractions to draw conclusions about certain special subfields.

5. Exercise 5 of Section 7.5.
   In this exercise you show that the field of fractions of the ring of formal power series $F[[x]]$ is the ring of Laurent series $F((x))$. These rings were the subject of two exercises in Section 7.2 that we solved on Homework 7 last quarter.

6. Exercises 3 and 4 of Section 7.6.
   These exercises show how the structure of ideals behaves under direct products.

7. Exercise 6 of Section 7.6.
   In this exercise you see an application of the Chinese Remainder Theorem to the study of polynomials in $\mathbb{Z}[x]$.