Some Notation

Sets and set notation

\{ \} A set
\(x \in \{2, 4, x, y, \text{apple}, z\}\) \(x\) is an element of the set consisting of 2, 4, \(x\), \(y\), \(z\) and an apple
\(y \notin \{2, 17, \text{green}\}\) \(y\) is not an element of the given set
\(\{x : x^2 = 5\}\) Set of \(x\) such that \(x^2 = 5\) (also written = \(\{x \mid x^2 = 5\}\))
\(\forall, \exists\) ‘for all’ and ‘there exists’
\(\emptyset\) The empty set
\(B \subseteq A\) \(B\) is a subset of \(A\): if \(x \in B\) then \(x \in A\)
\(A \setminus B\) ‘Setminus’: If \(B \subseteq A\) then \(A \setminus B\) is the set of elements of \(A\) that are not in \(B\).
\(\mathbb{Z}\) \(\{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}\) Set of integers
\(\mathbb{Z}^+\) or \(\mathbb{N}\) \(\{1, 2, 3, 4, \ldots\}\) Set of positive integers (natural numbers)
\(\mathbb{Z}^-\) \(\{\ldots, -3, -2, -1\}\) Set of negative integers
\(\mathbb{Z}_0^+\) or \(\mathbb{N}_0\) or \(\mathbb{W}\) \(\{0, 1, 2, 3, 4, \ldots\}\) Set of whole numbers
\(\mathbb{Q}\) \(\{p/q : p \in \mathbb{Z}, q \in \mathbb{N}\}\) Rational numbers (quotients)
\(\mathbb{Q}^+, \mathbb{Q}_0^+, \mathbb{Q}^-\) Positive, positive with zero & negative rational numbers respectively
\(\mathbb{Q}^*\) or \(\mathbb{Q}'\) Non-zero rational numbers
\(\mathbb{R}\) Set of real numbers
\(\mathbb{R}^+, \mathbb{R}^-, \mathbb{R}_0^+, \mathbb{R}^\times\) (or \(\mathbb{R}^*\)) Positive, negative, positive with zero and non-zero reals respectively
\(\mathbb{C}\) \(\{z = x + iy : x, y \in \mathbb{R}\} = \{re^{i\theta} : r \in \mathbb{R}_0^+, \theta \in [0, 2\pi)\}\) Complex numbers
\(\mathbb{R}^n\) \(n\)-dimensional real vector space
\(S^1\) \(\{x \in \mathbb{R}^2 \text{ (or } \mathbb{C}) : |x| = 1\}\) the unit circle in \(\mathbb{R}^2\) or \(\mathbb{C}\)
\(n\mathbb{Z}\) \(\{\ldots, -2n, -n, 0, n, 2n, \ldots\}\) set of multiples of \(n\)
\(|A|\) Cardinality of the set \(A\): number of elements in \(A\)
\(\aleph_0\) ‘Aleph-nought’: cardinality of any countably infinite set, e.g. \(\mathbb{Z}, \mathbb{Q}\)
\(2^{\aleph_0}\) Cardinality of sets \(\mathbb{R}, \mathbb{C}\): an uncountable infinity

Functions

Throughout, \(f : X \rightarrow Y\) is a function from the set \(X\) to the set \(Y\)

\(f : x \mapsto y\) \(f\) maps the element \(x \in X\) to the element \(y \in Y\): i.e. \(f(x) = y\)
\(\text{Im } f\) (or \(f(X)\)) Image of \(f\). I.e. \(\{y \in Y : y = f(x) \text{ for some } x \in X\}\)
\(\ker f\) Kernel of \(f\): if \(Y\) has an identity element \(e\), then \(\ker f = \{x \in X : f(x) = e\}\)
\(\cong\) Isomorphism (bijective homomorphism)
\(f^{-1}(Z)\) Inverse image: \(Z \subseteq Y \implies f^{-1}(Z) = \{x \in X : f(x) \in Z\}\). Thus \(\ker f = f^{-1}\{e\}\).

Relations

\(*\) Binary relation
\(\cdot\) Multiplication and addition (or binary relations written multiplicative/additively)
\(+, \cdot\) modulo \(n\) (usually \(n\) dropped): e.g. \(4 +_5 3 = 2\)
\(\circ\) Composition of functions, or of operators
### Matrices

- $M_{n}(\mathbb{R})$ | $n \times n$ matrices with real entries
- $I$ or $I_n$ | $n \times n$ identity matrix: 1’s down leading diagonal, 0’s elsewhere
- $\det A$ | Determinant of $A$
- $\text{tr } A$ | Trace of $A$: sum of entries down the leading diagonal
- $A^T$ | Matrix transpose
- $\text{GL}_n(\mathbb{R})$ | $n \times n$ matrices with $\det A \neq 0$: General linear matrices
- $\text{SL}_n(\mathbb{R})$ | $n \times n$ matrices with $\det A = 1$: Special linear matrices
- $\text{O}_n(\mathbb{R})$ | $\{ A \in M_{n}(\mathbb{R}) : AA^T = I \}$: Orthogonal matrices
- $\text{SO}_n(\mathbb{R})$ | $\{ A \in \text{SL}_n(\mathbb{R}) : AA^T = I \}$: Special orthogonal matrices
- $\text{SL}_n(\mathbb{Z})$ | $n \times n$ matrices with $\det A = 1$ and integer entries

### Groups

- $C_n$ | Cyclic group of order $n$
- $\mathbb{Z}_n$ | set of equivalence classes $\mathbb{Z}/\sim = \{[0],[1],\ldots,[n-1]\}$ of remainders modulo $n$
- $n\mathbb{Z}$ | Group of multiples of $n$ under $+$: i.e. $(n\mathbb{Z},+)$
- $S_n$ | Symmetric group on $n$ letters; permutations of $\{1,2,\ldots,n\}$ under composition
- $D_n$ | Dihedral group of order $2n$; symmetries of regular $n$-gon under composition
- $S^1$ | Unit circle in $\mathbb{C}$ under multiplication
- $H \leq G$ | $H$ a subgroup of $G$
- $G/H$ | Set of left cosets of $H$ in $G$
- $H < G$ | $H$ a proper subgroup of $G$
- $H \triangleleft G$ | $H$ a normal subgroup of $G$ ($gH = Hg, \forall g \in G$)
- $\langle x \rangle$ | Cyclic subgroup of $G$ generated by $x$