Math 13: Homework 6

Submit the following, and questions 6.3.4, 7.2.2, 7.2.4, and 7.3.4 at the discussion on Thursday 7th March

Let \( X = \mathbb{Z} \times \mathbb{N} \) and define a relation \( \sim \) on \( X \) by

\[
(a, b) \sim (c, d) \iff ad = bc
\]

Be careful! \( \sim \) is a relation on a set of pairs!

1. Show that \( \sim \) is an equivalence relation on \( X \). See if you can do this without writing any fractions!

2. Find the elements of the following equivalence classes:
   (a) \([2,3]\]
   (b) \([-4,10]\]

3. Define the function \( f : X \to \mathbb{Q} \) by \( f(x, y) = \frac{x}{y} \). Is the function \( f \) injective and/or surjective? Prove your assertions.

4. Prove that \( f(x, y) = f(v, w) \iff (x, y) \sim (v, w) \).