## Math 130A: Homework 1

Submit your answers to questions 1, 3, 4, 6, 7 & 8 at discussion on Wednesday 6th April

- 1. (a) How many different six-place license plates are possible if the first two places are for letters and the other four are for numbers?
  - (b) Repeat part (a) with the assumption that no letter or number can be repeated in a given license plate.
- 2. Fifteen workers are to be assigned to fifteen different jobs, one to each job. How many different assignments are possible?
- 3. (a) In how many ways can 4 boys and 3 girls sit in a row?
  - (b) What if the the boys are to sit together and so are the girls?
  - (c) What if only the boys must sit together?
  - (d) What if no two people of the same sex can sit together?
- 4. A child has 12 blocks, or which 5 are black, 4 are red, 2 are white and 1 blue. If the child puts the blocks in a line, how many arrangements are possible?
- 5. How many five-card poker hands are there?
- 6. Expand  $(x^2 + y + 1)^4$ .
- 7. Prove the following identities:

(a) 
$$\binom{n}{r-1} + \binom{n}{r} = \binom{n+1}{r}$$
 for all  $1 \le r \le n$ 

(b) 
$$\sum_{r=0}^{n} {n \choose r} = 2^{n}$$
  
(c)  ${n+m \choose r} = \sum_{k=0}^{r} {n \choose k} {m \choose r-k}$   
(d)  ${2n \choose n} = \sum_{k=0}^{n} {n \choose k}^{2}$ 

8. From a set of *n* people, a committee of size *k* with a committee chair are to be chosen.

- (a) Explain why there are  $k\binom{n}{k}$  ways to do this.
- (b) By first choosing the chair, and then the remainder of the committee, establish the identity

$$\sum_{k=1}^{n} k \binom{n}{k} = n2^{n-1}$$