## Math 130A: Homework 1

Submit your answers to questions $1,3,4,6,7 \& 8$ at discussion on Wednesday 6th April

1. (a) How many different six-place license plates are possible if the first two places are for letters and the other four are for numbers?
(b) Repeat part (a) with the assumption that no letter or number can be repeated in a given license plate.
2. Fifteen workers are to be assigned to fifteen different jobs, one to each job. How many different assignments are possible?
3. (a) In how many ways can 4 boys and 3 girls sit in a row?
(b) What if the the boys are to sit together and so are the girls?
(c) What if only the boys must sit together?
(d) What if no two people of the same sex can sit together?
4. A child has 12 blocks, or which 5 are black, 4 are red, 2 are white and 1 blue. If the child puts the blocks in a line, how many arrangements are possible?
5. How many five-card poker hands are there?
6. Expand $\left(x^{2}+y+1\right)^{4}$.
7. Prove the following identities:
(a) $\binom{n}{r-1}+\binom{n}{r}=\binom{n+1}{r}$ for all $1 \leq r \leq n$
(b) $\sum_{r=0}^{n}\binom{n}{r}=2^{n}$
(c) $\binom{n+m}{r}=\sum_{k=0}^{r}\binom{n}{k}\binom{m}{r-k}$
(d) $\binom{2 n}{n}=\sum_{k=0}^{n}\binom{n}{k}^{2}$
8. From a set of $n$ people, a committee of size $k$ with a committee chair are to be chosen.
(a) Explain why there are $k\binom{n}{k}$ ways to do this.
(b) By first choosing the chair, and then the remainder of the committee, establish the identity

$$
\sum_{k=1}^{n} k\binom{n}{k}=n 2^{n-1}
$$

