Math 130A: Homework 2

Submit your answers to questions 2, 5, 6, 7, 8 & 9 on Gradescope by Wednesday 13$^{\text{th}}$ April

1. A die is rolled continually until a 6 appears, at which point the experiment stops.
   (a) What is the sample space for this experiment?
   (b) If E_n is the event that n rolls are required to complete the experiment, what is the set E_n?
   (c) What is $\bigcup_{n=1}^{\infty} E_n$?

2. Suppose that A and B are mutually exclusive events for which $\mathbb{P}(A) = 0.3$ and $\mathbb{P}(B) = 0.5$. What is the probability that
   (a) Either A or B occurs?
   (b) Both A and B occur?
   (c) A occurs but B does not?

3. Fifteen members of an adult soccer team are asked their type of job (blue collar or white collar) and political affiliation (Republican, Democratic or Independent). How many outcomes are
   (a) In the sample space?
   (b) In the event that at least one of the team members is a blue-collar worker?
   (c) In the event that none of the team members consider themselves an Independent?

4. Suppose all $\binom{52}{5}$ possible poker hands are equally likely of being dealt. Find the probability of being dealt
   (a) A flush? (all 5 cards of the same suit)
   (b) One pair (a hand of the form a,a,b,c,d where a,b,c,d have distinct values)
   (c) Two pairs (a,a,b,b,c where a,b,c have distinct values)
   (d) Three of a kind?
   (e) Four of a kind?

5. A pair of fair dice is rolled and the sum is computed.
   (a) Find the probability that the sum equals n for each $n = 2, 3, 4, \ldots, 12$.
   (b) The dice are rolled until a sum of either 4 or 9 appears. Find the probability that 4 appears first.

6. A forest contains 30 elk, of which 8 are captured, tagged and then released. Some weeks later, 5 of the elk are captured. Supposing that the elk are tagged and caught with equal likelihood, what is the probability that exactly 3 of these elk have been tagged?

7. Suppose that E and F are two events. Prove that the probability of *exactly one* of these events occurring is

$$\mathbb{P}(E) + \mathbb{P}(F) - 2\mathbb{P}(E \cap F)$$

8. An urn contains m white and n black balls. If a random sample of size r is chosen, what is the probability that it contains exactly $k \leq r$ white balls?

9. Consider an experiment whose sample space consists of a countably infinite number of points $S = \{x_1, x_2, x_3, \ldots\}$. Show that not all points can be equally likely. Can all points have a positive probability of occurring?

10. Use induction to prove the second Bonferroni inequality

$$\mathbb{P}\left(\bigcup_{i=1}^{n} E_i\right) \geq \sum_{i=1}^{n} \mathbb{P}(E_i) - \sum_{i<j} \mathbb{P}(E_i \cap E_j)$$

(*Hint: use the 1st Bonferroni inequality in the induction step!*)