Math 130A: Homework 4

Submit your answers to questions 1, 3, 5, 6, 7 & 8 on Gradescope (in Canvas) by Wednesday 11th May

1. Suppose a fair die is rolled twice. For each of the following random variables X, state the possible values of X.

    (a) The maximum value to appear in the two rolls.
    (b) The minimum value to appear in the two rolls.
    (c) The sum of the two rolls.
    (d) The value of the first roll minus the value of the second.

2. Two fair dice are rolled. Let X be the product of the two values. Compute the probability mass function of X. What is its expectation?

    (*Use a spreadsheet if you like!*)

3. Let X be the winnings of a gambler. Let $p(i) = \mathbb{P}\{X = i\}$ and suppose that

    $$p(0) = \frac{1}{3}; \quad p(1) = p(-1) = \frac{13}{55}; \quad p(2) = p(-2) = \frac{1}{11}; \quad p(3) = p(-3) = \frac{1}{165}$$

    Compute the conditional probability that the gambler wins $i = 1, 2, 3$ given that he wins a positive amount.

4. An integer n is selected at random from the set $\{1, 2, 3 \ldots, 10^3\}$ (each integer has equal probability 10^{-3}). What is the probability that n will be divisible by 3? By 5? By 7? By 15? By 105? How does the answer change if 10^3 is replaced by 10^k as k becomes larger?

5. Two coins are flipped. The first lands heads with probability 0.4 and the second with probability 0.8. Assume the results of the flips are independent.

    (a) Find $\mathbb{P}\{X = 1\}$.
    (b) Determine $\mathbb{E}[X]$.

6. A person tosses a fair coin until a tail appears for the first time. If the tail appears on the nth flip, the person wins $\$2^n$. Let X denote the player's winnings.

    (a) Show that $\mathbb{E}[X] = \infty$
    (b) Compute $\mathbb{P}\{X \geq \$1 \text{ million}\}$. Would you be willing to pay $1 million to play this game once?
    (c) Would you be willing to pay $1 million for each game if you could play for as long as you liked and only had to settle up when you wanted to stop playing?

7. If $\mathbb{E}[X] = -3$ and Var $X = 5$, find

    (a) $\mathbb{E}[(2 + X)^2]$;   (b) $\text{Var}(4 + 3X)$

8. A box contains 4 red and 5 blue marbles. Two marbles are withdrawn randomly. If they are the same color, you win $12. If they are different colors, you lose $10. Calculate;

    (a) The expected amount you win/lose.
    (b) Its variance.