Math 130A: Homework 5

Submit your answers to questions 1, 3, 4, 6, 9 & 10(a) on Gradescope by Wednesday 18th May

1. A man claims to have extrasensory perception (ESP). As a test, a fair coin is flipped 10 times and the man is asked to predict the outcome in advance. He gets 7 out of 10 correct. What it the probability that he would have done at least this well if he did not have ESP?

2. It takes 9 votes from a 12-person jury to convict a defendant. Suppose that the probability of a juror voting a guilty person innocent is 0.2, and an innocent person guilty is 0.1. If each juror acts independently and if 65% of defendants are guilty, find the probability that a jury renders the correct verdict.

3. When coin 1 is flipped if lands heads with probability 0.4; when coin 2 is flipped it lands heads with probability 0.7. One of the coins is chosen randomly and flipped 10 times.
   (a) Find the probability that the coin lands heads on exactly 7 of the 10 flips.
   (b) Given that the first of the 10 flips lands heads, what is the conditional probability that exactly 7 of the 10 flips will be heads?

4. Suppose that the average number of cars abandoned on a certain highway is 2.2 per week. Approximate the probability that there will be
   (a) No abandoned cars in the next week;
   (b) At least 2 abandoned cars in the next week.

5. Compare the Poisson approximation with the correct binomial probability for the following cases:
   (a) $\mathbb{P}\{X = 2\}$ when $n = 8, p = 0.1$;
   (b) $\mathbb{P}\{X = 9\}$ when $n = 10, p = 0.95$;
   (c) $\mathbb{P}\{X = 0\}$ when $n = 10, p = 0.1$;
   (d) $\mathbb{P}\{X = 4\}$ when $n = 9, p = 0.2$.

6. A fair coin is repeatedly flipped until heads appears for the 10^{th} time. Let X be the number of tails that occur. Compute the probability mass function of X.

7. If X has cumulative distribution function F, what is the distribution function of e^X?

8. Suppose X has mean μ and variance σ^2. Find the mean and variance of $Y = \frac{X-\mu}{\sigma}$.

9. Let $X \sim B(n, p)$. If $q = 1 - p$, prove that
$$\mathbb{E}\left[\frac{1}{X+1}\right] = \frac{1-q^{n+1}}{(n+1)p}$$

10. Suppose $X \sim \text{Poisson}(\lambda)$.
    (a) Show that $\mathbb{E}[X^n] = \lambda \mathbb{E}[(X+1)^{n-1}]$ and use this to compute $\mathbb{E}[X^3]$.
    (b) Compute $\mathbb{E}\left[\left(\frac{X-\mu}{\sigma}\right)^3\right]$. Why are you not surprised at the *sign* of this value?