Math 130B Homework 1

Hand in the following questions at discussion on Thursday 14th January

1. Two fair dice are rolled. Find the joint probability mass function of X and Y when:
   (a) X is the largest value obtained on any die and Y is the sum of the values;
   (b) X is the value on the first die and Y is the larger of the two values;
   (c) X is the smallest and Y is the largest value obtained on the dice.

   Give your answer both formulaically and in tabular form. Sum the rows and columns of your tables to check that you recover the probability mass functions for X and Y.

2. The joint probability density function of X and Y is given by
   $$f(x,y) = c(y^2 - x^2)e^{-y}, \qquad -y \le x \le y, \quad 0 < y < \infty$$
   (a) Find the constant c.
   (b) Find the marginal density of Y.
   (c) Find the marginal density of X (first think about what the *domain* of y is given a fixed value of x: this is what you have to integrate over…)
   (d) Find $\mathbb{E}(X)$.

3. The joint density function of X and Y is given by
   $$f(x,y) = \frac{6}{7}\left(x^2 + \frac{xy}{2}\right), \qquad 0 < x < 1, \quad 0 < y < 2$$
   (a) Verify that this is indeed a joint density function.
   (b) Compute the density function of X.
   (c) Find $\mathbb{P}(X > Y)$.
   (d) Find $\mathbb{P}(Y > \frac{1}{2} | X < \frac{1}{2})$.
   (e) Find $\mathbb{E}(X)$.
   (f) Find $\mathbb{E}(Y)$.

4. Suppose that n points are independently chosen at random on the circumference of a circle and we want the probability that they all lie in some semicircle. That is, we want the probability that there is a line passing through the center of the circle such that all points are on one side of that line.
   Let $P_1, P_2, \ldots, P_n$ be the n points. Let A denote the event that all the points are contained in some semicircle, and let A_i be the event that all the points lie in the semicircle beginning at P_i and going clockwise for[1] $180°$.
   (a) Express A in terms of the A_i.
   (b) Are the A_i mutually exclusive?
   (c) Find $\mathbb{P}(A)$.

[1] To make this more precise, if P_i has polar θ, then the semicircle in question consists of those points with polar angle lying in the *half-open interval* $[\theta, 180 + \theta)$. Any finite collection of points lying in such a semicircle will be on one side of some line. It isn't relevant to the computation of probabilities that the interval be half-open, given that the distribution of the points P_i on the circle is continuous.