Math 130B Homework 2

Hand in the following questions at discussion on Thursday 21st January

1. The joint density function of X and Y is

$$\begin{cases} x+y & \text{if } 0 < x < 1 \text{ and } 0 < y < 1 \\ 0 & \text{otherwise} \end{cases}$$

   (a) Are X and Y independent?
   (b) Find the density function of X.
   (c) Find $\mathbb{P}(X+Y<1)$ using a double integral.
   (d) (Hard!) For the purposes of this part of the question, we assume that X and Y are independent, and define their joint density function to be $g_{X,Y}(x,y) = f_X(x)f_Y(y)$. Find the cumulative distribution function $G_{X+Y}(a)$ using convolutions. Draw sketches to help you compute the necessary integrals and sketch your answer.

2. Suppose that 10^6 people arrive at a service station at times that are independent random variables, each of which is uniformly distributed over $(0, 10^6)$. Let N denote the number that arrive in the first hour. Find an approximation to $\mathbb{P}(N=i)$.

3. Suppose that A, B and C are independent random variables, each being uniformly distributed over $(0,1)$.
   (a) What is the joint cumulative distribution function of A, B and C?
   (b) What is the probability that all the roots of the equation $Ax^2+Bx+C=0$ are real?

4. If X and Y are jointly continuous with joint density function $f_{X,Y}(x,y)$, show that $X+Y$ is continuous with density function

$$f_{X+Y}(t) = \int_{-\infty}^{\infty} f_{X,Y}(x, t-x)\,\mathrm{d}x$$

   Note: the result in class required X, Y to be independent. Here we do not assume this!