Math 130B Homework 3

Hand in the following questions at discussion on Thursday 28th January

1. Jill's bowling scores are apprixmately normally distributed with mean 170 and standard deviation 20, while Jack's scores are approximately normally distributed with mean 160 and standard deviation 15. If Jack and Jill each bowl one game, then assuming that their scores are independent random variables, approximate the probability that:

   (a) Jack's score is higher.

   (b) The total of their scores is above 350.

2. The joint probability mass function of X and Y is given by

$$p(1,1) = \frac{1}{8} \qquad p(1,2) = \frac{1}{4} \qquad p(2,1) = \frac{1}{8} \qquad p(2,2) = \frac{1}{2}$$

   (a) Compute the conditional mass function of X given that $Y = i$ for $i = 1, 2$.

   (b) Are X and Y independent?

   (c) Compute $\mathbb{P}(XY \leq 3)$, $\mathbb{P}(X + Y > 2)$ and $\mathbb{P}(\frac{X}{Y} > 1)$.

3. Suppose that Oil is currently at \$30 per barrel and its price ratio over the next year is expected to be lognormally distributed: its price this time next year will be $O = \$30e^X$ where $X \sim N(-0.2, 0.8)$. Similarly, Drillcor's stock is currently at \$50/share and its year on year growth forecast is lognormally distributed: $D = \$50e^Y$ where $Y \sim N(-0.1, 0.4)$. Suppose that the correlation of the random variables X, Y is $\rho = 0.5$.

   (a) Compute $\mathbb{E}(O)$ and $\mathbb{E}(D)$.

   (b) Find the probability that Oil will at least double in price over the coming year. Do the same thing for Drillcor.

   (c) Suppose that $X = \ln 2$ so that the oil price has doubled to \$60. What are the mean and variance of the conditional distribution $Y|X$?

   (d) Compute $\mathbb{E}(D|O = \$60)$ and the probability $\mathbb{P}(D > \$100|O = \$60)$.

   (e) Repeat your calculations for part (d) with correlations $\rho = 0.7, 0.8, 0.9, 0.99$. Can you explain why the probability $\mathbb{P}(D > \$100|O = \$60)$ eventually starts to *decrease* for higher correlations ρ?

   (f) (Optional) Find the value of ρ for which $\mathbb{P}(D > \$100|O = \$60)$ is maximal.