Math 161: Homework 2

Submit your answers to the starred questions at the discussion class on Thursday 24th January

1. * Explain why Angle-Angle-Angle (AAA) and Side-Side-Angle (SSA) are not triangle congruence theorems.

2. (a) Two lines meeting at a point create two pairs of ‘vertical’ angles. Prove that vertical angles are congruent using Euclid’s postulates.  
   (Hint: This is one place where you will need to use postulate 4 regarding right angles . . .)  
   (b) Use part (a) to complete Euclid’s proof of the Exterior Angle Theorem (I. 16) as shown in the notes for chapter 1. That is, explain why $\beta < \delta$.

3. Use Hilbert’s axioms C-4 and C-5 to prove that congruence of angles is an equivalence relation.

4. * Show that if two sides of a triangle are not congruent, then the angles opposite those sides are not congruent.  
   (Hint: Use isosceles triangles and ASA)

5. * Prove that Playfair’s Postulate is equivalent to the following statement: Whenever a line is perpendicular to one of two parallel lines, it must be perpendicular to the other.

6. * Suppose we are working in Hilbert’s geometry and that you are given two distinct points $A, B$. Carefully prove the existence of a point $F$ lying between $A$ and $B$. In particular:
   (a) Use the picture to help you construct or show the existence of each of the points $C, D, E, F$ in alphabetical order. You should only need Hilbert’s axioms up to and including O-4, and the theorem that the intersection of distinct lines is unique (follows from I-2).
   (b) Explain why $D$ does not lie on $AB$.
   (c) Explain why $E$ does not lie on $\triangle ABD$.
   (d) Explain why $E \neq C$ (so that $\overrightarrow{CE}$ exists).
   (e) Explain why $F$ lies on $\overline{AB}$ and not on $BD$.

7. Draw pictures to help illustrate as many of Hilbert’s axioms as you can.