Math 2A Single Variable Calculus Homework Questions Chapter 1

1 Functions and Models

1.1 Four Ways to Represent a Function

1. Temperature readings T (in °F) were recorded every two hours from midnight to 2pm. Time t was measured in hours from midnight.

t	0	2	4	6	8	10	12	14
T	82	75	74	75	84	90	93	94

   (a) Use the readings to sketch a rough graph of T as a function of t.

   (b) Use your graph to estimate the temperature at 9pm.

2. A spherical baloon with radius r inches has volume $V(r) = \frac{4}{3}\pi r^3$. Find a function that represents the amount of air required to inflate the baloon from a radius of r inches to a radius of $r+1$ inches.

3. Find the domain of the function $f(x) = \frac{x+4}{x^2-9}$.

4. Find the domain of the function $f(x) = \frac{2x^3-5}{x^2+x-6}$.

5. Find the domain of the function $f(u) = \frac{u+1}{1+\frac{1}{u+1}}$.

6. Find the domain and sketch the graph of the function

$$f(x) = \begin{cases} 3 - \frac{1}{2}x & \text{if } x \leq 2, \\ 2x - 5 & \text{if } x > 2. \end{cases}$$

7. Find the domain and sketch the graph of the function

$$f(x) = \begin{cases} x + 9 & \text{if } x < -3, \\ -2x & \text{if } |x| \leq 3, \\ -6 & \text{if } x > 3. \end{cases}$$

8. A box (without lid) is to be made by cutting squares of side-length $x$in from the corners of a piece of card which is 12in by 20in and folding up the edges. Find the volume V of the box as a function of x.

9. A cell phone plan has a basic charge of \$35 per month and includes 400 free minutes and charges 10 cents per additional minute. Find and graph the monthly cost of the plan C as a function of the number of used minutes x for $0 \leq x \leq 600$.

1.2 Mathematical Models: A Catalog of Essential Functions

1. What do all members of the family of linear functions $f(x) = 1 + m(x+3)$ have in common? Sketch several members of the family.

2. The average surface temperature of the earth is modeled by $T = 0.02t + 8.50$ where T is the temperature in °C and t represents years since 1900.

   (a) What do the slope and T-intercept represent?

   (b) Use the equation to predict the average global surface temperature in 2100.

3. The relationship between the Fahrenheit (F) and Celcius (C) temperature scales is given by the linear function $F = \frac{9}{5}C + 32$.

   (a) Sketch a graph of this function.

   (b) What is the slope of the graph and what does it represent? What is the F-intercept and what does it represent?

4. Many physical quantities are connected by *inverse square laws,* that is, by power functions of the form $f(x) = kx^{-2}$, where k is constant. I.e. the illumination of an object by a light source is inversely poroportional to the square of the distance from the source. Imagine after dark you are reading a book illuminated by a single light which is too dim. You move halfway towards the light. How much brighter is the lamp?

5. Ecologists have modeled the species-of-bat-per-unit-area relationship with a power function $S = 0.7A^{0.3}$, where S is the number of species living in an area A.

   (a) If a cave has area 60m^2, how many species would you expect to find in the cave?

   (b) If only four species of bat live in a cave, estimate the area of the cave.

1.3 New Functions from Old Functions

1. Graph the function $y = (x-1)^3$ by transforming the graph of a standard function.

2. Graph the function $y = 4\sin 3x$ by transforming the graph of a standard function.

3. Graph the function $y = 1 - 2\sqrt{x+3}$ by transforming the graph of a standard function.

4. A variable star has time between periods of maximum brightness of 5.4 days, average brightness 4.0 and the brightness varies by ± 0.35 magnitude. Find a function which models the brightness as a function of time.

5. Find the functions $f \circ g$, $g \circ f$, $f \circ f$ and $g \circ g$ and their domains for the following pairs of functions: $f(x) = 1 - 3x$, $g(x) = \cos x$.

6. Find the functions $f \circ g$, $g \circ f$, $f \circ f$ and $g \circ g$ and their domains for the following pairs of functions: $f(x) = \sqrt{x}$, $g(x) = \sqrt[3]{1-x}$.

7. A spherical balloon is being inflated and the radius is increasing at a rate of 2cm/s.

   (a) Express the radius r of the balloon as a function of the time t in seconds.

   (b) If V is the volume of the balloon as a function of the radius, find $V \circ r$ and interpret it.

1.4/5 Exponential Functions

1. Use the law of Exponents to rewrite and simplify the expressions:

   (a) $8^{4/3}$, (b) $x(3x^2)^3$.

2. Starting with the graph of $y = e^x$, find the equation of the graph that results from

   (a) Reflecting about the line $y = 4$.

   (b) Reflecting about the line $x = 2$.

3. Find the domain of each function:

   (a) $g(t) = \sin(e^{-t})$, (b) $g(t) = \sqrt{1 - 2^t}$.

1.5/6 Inverse Functions and Logarithms

1. Is the function $f(x) = 10 - 3x$ 1–1? What about $g(x) = \cos x$? Justify your answers.

2. If $f(x) = x^5 + x^3 + x$, find $f^{-1}(3)$ and $f(f^{-1}(2))$.

3. Find a formula for the inverse of the function $f(x) = \frac{4x-1}{2x+3}$.

4. Find the exact values of the expressions

   (a) $e^{-2\ln 5}$, (b) $\ln(\ln(e^{e^{10}}))$.

5. When a camera flash goes off, the batteries immediately begin to recharge the flash's capacitor, which stores charge given by

   $$Q(t) = Q_0(1 - e^{-t/a}).$$

   (The maximum charge capacity is Q_0 and t is measured in seconds.)

   (a) Find the inverse of this function and explain its meaning.

   (b) How long does it take to recharge the capacitor to 90% of capacity if $a = 2$?

6. Simplify the expression $\cos(2\tan^{-1} x)$.