1 Functions and Models

No questions from ch 1 in final. Just for review: all this should be familiar. If not, study!

1.1 Four Ways to Represent a Function

Interval notation Should be familiar with notation for different types of interval. All intervals are subsets of the real numbers $\mathbb{R} = (-\infty, \infty)$. For example:

Open: $(-3, 5) = \{x \in \mathbb{R} : -3 < x < 5\}$. All numbers *strictly between* -3 and 5.

Closed: $[-3, 5] = \{x \in \mathbb{R} : -3 \leq x \leq 5\}$. All numbers *between and including* -3 and 5.

Half-open: $[-3, 5) = \{x \in \mathbb{R} : -3 \leq x < 5\}$. Includes one but not the other endpoint. Could also have $(-3, 5]$.

We use the union symbol $\cup$ to join together two or more intervals to make a larger set: e.g.

$$(1, 3) \cup [7, 9) = \{x \in \mathbb{R} : 1 < x < 3 \text{ or } 7 \leq x < 9\}$$

Definition. *A* function *f is a rule that assigns to each element x in a set D exactly one element $f(x)$ in a set E.*
The domain *of f is the set D*
The range *of f is a subset of E consisting of the values attained by applying f to all of the values in the domain:*

$$\text{range}(f) = \{f(x) : x \in D\} \subseteq E$$

x is the independent variable
$y = f(x)$ is the dependent variable

In this class, both domain and range will always be subsets of the real numbers $\mathbb{R}$.

Representations We may describe a function in several ways:

1. Verbally E.g. 'Take the integer part'
2. Numerically Give a table of values
3. Visually Draw a graph
4. Algebraically $f(x) = \lfloor x \rfloor$

Should be able to convert between these descriptions and find domains of functions from formulas and graphs.

Example The function $f(x) = \sqrt{x^2 - 4}$ has domain $(-\infty, -2] \cup [2, \infty)$.

Vertical Line test A curve in the xy-plane is the graph of a function if and only if each vertical line intersects the curve no more than once.

Piecewise functions Any function where multiple formuæ are required to describe the function. For example

$$f(x) = \begin{cases} x^2 + 1 & \text{if } x < 1 \\ 4 - x & \text{if } x \geq 1 \end{cases}$$

Symmetry Let a be a positive real number and I be one of the symmetric intervals $(-a, a)$ or $[-a, a]$. We say that:

- f is *even* on I if, for all $x \in I$, we have $f(-x) = f(x)$.
- f is *odd* on I if, for all $x \in I$, we have $f(-x) = -f(x)$.

Increasing and Decreasing Functions

Definition. *f is* increasing *on an interval I if,*

For all $x_1, x_2 \in I$, we have that $x_1 < x_2 \implies f(x_1) < f(x_2)$

Decreasing is similar: $x_1 < x_2 \implies f(x_1) > f(x_2)$.

Homework

1. Consider the function $f(x) = \frac{1}{x^2-1}$.
   (a) Suppose that the domain of f is chosen to be the set of real numbers x for which the formula $f(x)$ is defined. What is the domain?
   (b) Find all the intervals on which f is increasing or decreasing. Prove your assertions.
2. Suppose that f and g are both even functions.
   (a) Decide whether the functions $f + g$ and fg are even, odd or neither. *Prove* your assertions.
   (b) Repeat part (a) in the situation that f and g are both odd.
   (c) What if f is odd and g even?