1.2 Mathematical Models: a catalogue of essential functions

Should be familiar with the following types of functions.

Linear Graphs are straight lines. Equations $y = mx + c$ where m is the gradient and c the y-intercept.

Power Equations of the form $f(x) = x^a$, where a is a constant. If a is an even/odd integer, so is the function f. If a is a fraction, then the power function contains a root. For example

$$f(x) = x^{7/4} = (\sqrt[4]{x})^7 = \sqrt[4]{x^7}$$

Polynomial Equations of the form $p(x) = a_n x^n + \cdots + a_1 x + a_0$. The *degree* is the highest power (e.g. n). The constants a_k are the *coefficients.* Examples include quadratic, cubic, quartic functions.

Rational Equations of the form $f(x) = \dfrac{p(x)}{q(x)}$ where both p and q are polynomials.

Algebraic (In this class) Functions whose formula is built entirely from the basic algebraic operations: adding, subtracting, multiplying, dividing, raising to (fractional) powers. For example

$$f(x) = \frac{17x^{3/2} - \sqrt{1+3x^4}}{19x^3(x+7)^{13/8}}$$

Trigonometric Sine, Cosine, Tangent, Secant, Cosecant, Cotangent and their combinations. You should know the graphs and special values of the first three, and be able to compute/graph the co-functions from their definitions:

$$\sec x = \frac{1}{\cos x}, \qquad \csc x = \frac{1}{\sin x}, \qquad \cot x = \frac{1}{\tan x} = \frac{\cos x}{\sin x}$$

You should also know the basic identities.

- $\sin^2 x + \cos^2 x = 1$ and the tan/sec and cot/csc equivalents.
- Double angle formulæ for $\sin 2x$ and $\cos 2x$.
- Multiple angle formulæ for $\sin(x \pm y)$ and $\cos(x \pm y)$.

Exponential and logarithmic functions will be dealt with later.

Homework

1. Give a example formula for an algebraic function which is always positive, and has domain $[2, 6) \cup (6, 10]$. There are many possible answers.
2. Let m, n be positive integers with no common factors and let $f(x) = x^{m/n}$.
   (a) What is the domain of f? Your answer should depend on n.
   (b) Find all combinations of m, n for which f is an even function. Prove your assertion.
   (c) Repeat part (b) for when f is odd.