Math 2B Single Variable Calculus
Midterm 1 (v1)
Fall 2016

Name:

Student Id#:

Discussion Class Time:

Total marks = 50 (per question in brackets)
No calculators or other electronic devices
Unless otherwise stated, include all your working for full credit
Try all parts of every question, even if you can't do the first part

Question	Marks
1	/14
2	/6
3	/7
4	/15
5	/8
Total	/50

1. Evaluate the following integrals:

   (a) $\int_0^{\pi} 3x^2 + \sin x \, \mathrm{d}x$ (3)

   (b) $\int \frac{2x}{\sqrt{1+x^2}} \, \mathrm{d}x$ (3)

(c) $\int (x + x^{-1})^2 \, \mathrm{d}x$ (4)

(d) $\int_1^e \frac{1}{x} (\ln x)^2 \, \mathrm{d}x$ (4)

2. Evaluate the following:

(a) $\dfrac{\mathrm{d}}{\mathrm{d}x}\displaystyle\int_4^{x^2} \sin(\cos t)\,\mathrm{d}t$ (3)

(b) $\dfrac{\mathrm{d}}{\mathrm{d}y}\displaystyle\int_y^{6} x^3 - 3^x\,\mathrm{d}x$ (3)

3. Consider the Riemann sum

$$\sum_{i=1}^{8} \left(1 + \frac{i}{4}\right)^3 \frac{1}{4}$$

(a) Find a function f and an interval $[a, b]$ such that the Riemann sum approximates the area under the curve $y = f(x)$ between $x = a$ and $x = b$. (4)

(b) Evaluate the area in described in part (a). (3)

4. The region R lies between the curves $y = 3 - x^2$, $y = 4$. $x = 0$ and $x = 1$.

   (a) Compute the area of R. (4)

   (b) Compute the volume when R is rotated around the x-axis. (5)

(c) Compute the volume when R is rotated around the y-axis.
Hint: one integral isn't enough! (6)

5. A particle has acceleration $a(t) = 1 - e^{-t}\,\mathrm{m/s^2}$ (meters per second squared) at t seconds. Suppose that the initial velocity of the particle is $v_0 = 2\,\mathrm{m/s}$ and its initial position is $x_0 = 0\,\mathrm{m}$. Find how far the particle has travelled after 2 s.
   Hint: Recall that $a(t) = \frac{\mathrm{d}^2x}{\mathrm{d}t^2}$ and be careful with your constants of integration! (8)