Chapter 10: Written Problems

These problems are mostly from the textbook. The first few are typically more representative of exam/quiz questions than what you'll find in WebAssign. The last question(s) are more challenging. Solutions will automagically appear within Canvas. Some of these are trivial if you use a graphing device/computer, but can you do them *without*...?

10.1 Curves Defined by Parametric Equations

1. Sketch the curve by using the parametric equations to plot points. Indicate with an arrow the direction in which the curve is traced as t increases.

   (a) $x = t^2, \quad y = t^3 - 4t, \quad -3 \le t \le 3$

   (b) $x = e^{-t} + t, \quad y = e^t - t, \quad -2 \le t \le 2$

2. Sketch each curve parametrized by x and y, indicating the direction of travel. When the domain of the parameter is not given, assume that it is as large as possible. Finally, eliminate the parameter to find a Cartesian equation of the curve.

   (a) $x = 1 - 2t, \quad y = \frac{1}{2}t - 1, \quad -2 \le t \le 4.$

   (b) $x = t - 1, \quad y = t^3 + 1, \quad -2 \le t \le 2.$

   (c) $x = \frac{1}{2}\cos\theta, \quad y = 2\sin\theta, \quad 0 \le \theta \le \pi.$

   (d) $x = \sqrt{t+1}, \quad y = \sqrt{t-1}.$

3. Match the parametric equations with the graphs labeled I–VI. Give reasons for your choice.

   (a) $x = t^4 - t + 1, \quad y = t^2$

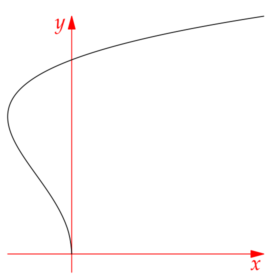
   (b) $x = t^2 - 2t, \quad y = \sqrt{t}$

   (c) $x = \sin 2t, \quad y = \sin(t + \sin 2t)$

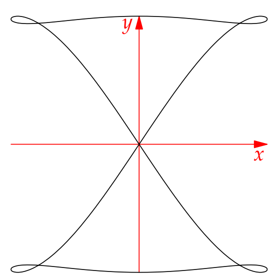
   (d) $x = \cos 5t, \quad y = \sin 2t$

   (e) $x = t + \sin 4t, \quad y = t^2 + \cos 3t$

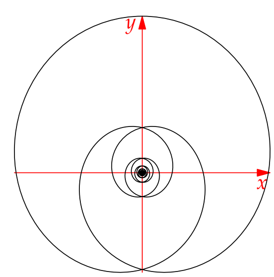
   (f) $x = \dfrac{\sin 2t}{4 + t^2}, \quad y = \dfrac{\cos 2t}{4 + t^2}$



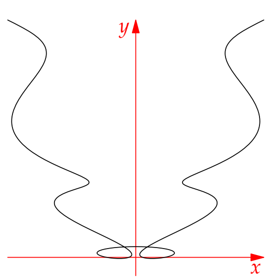
I



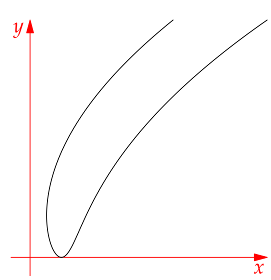
II



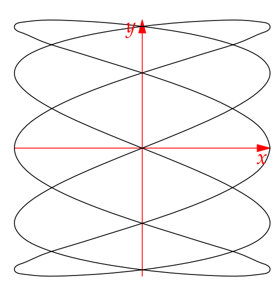
III



IV



V



VI

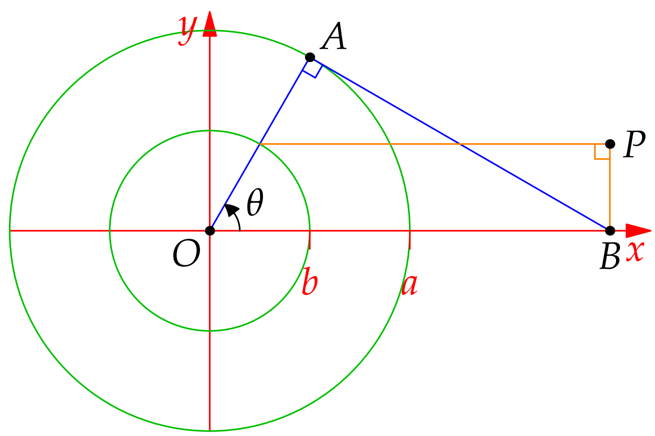
4. Let P be a point at a distance d from the center of a circle of radius r. The curve traced out by P as the circle rolls along the straight line is called a *trochoid*. The cycloid is the special case where $d = r$. Supposing that $t = 0$ when P is at its lowest point, where t is the same angle used in our description of the cycloid, prove that the parametric equations of the trochoid are

$$x(t) = rt - d\sin t, \qquad y = r - d\cos t$$

5. If a and b are fixed numbers, find parametric equations for the curve that consists of all possible positions of the point P in the figure, using the angle θ as the parameter. The line segment AB is tangent to the larger circle. Show further, by eliminating the parameter, that the Cartesian equation of the curve is

$$\frac{a^2}{x^2} + \frac{y^2}{b^2} = 1$$

Finally, sketch the curve!



10.2 Calculus with Parametric Curves

1. Find an equation of the tangent line to the curve parametrized by $x = t - t^{-1}$ and $y = 1 + t^2$, at $t = 1$.

2. Repeat question 1. for $x = \sin^3\theta, \quad y = \cos^3\theta, \quad \theta = \frac{\pi}{6}$.

3. A curve is parametrized by $x = \cos t + \cos 2t, \quad y = \sin t + \sin 2t$. Find the equation of the tangent(s) to the curve at the point $(-1, 1)$. Also graph the curve and the tangents.

4. Let $x = t^3 + 1, \quad y = t^2 - t$. Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. For which values of t is the curve concave up?

5. A curve is parametrized by $x = t^3 - 3t, \quad y = t^3 - 3t^2$. Find the points on the curve where the tangent is horizontal or vertical.

6. Find equations of the tangents to the curve $x = 3t^2 + 1, \quad y = 2t^3 + 1$ that pass through the point $(4, 3)$.

7. Find the area enclosed by the curve $x = t^2 - 2t, \quad y = \sqrt{t}$ and the y-axis.

8. Find the exact length of the curve defined by $x = e^t + e^{-t}$, $\quad y = 5 - 2t$, where $0 \le t \le 3$.

9. Find the exact length of the curve defined by $x = 3\cos t - \cos 3t$, $\quad y = 3\sin t - \sin 3t$, where $0 \le t \le \pi$.

10.3 Polar Co-ordinates

1. Sketch the region in the plane consisting of points whose polar co-ordinates satisfy the given conditions.

   (a) $0 \le r < 2, \quad \pi \le \theta \le \frac{3\pi}{2}$

   (b) $1 \le r \le 3, \quad \frac{\pi}{6} < \theta < \frac{5\pi}{6}$

   (c) $r \ge 1, \quad \pi \le \theta \le 2\pi$

2. Identify the curve by finding a Cartesian equation for the curve.

   (a) $r = 4\sec\theta$

   (b) $r = \tan\theta\sec\theta$

3. Find a polar equation for the curve represented by the given Cartesian equation.

   (a) $y = x$

   (b) $xy = 4$

4. Sketch the curve with the given polar equation.

   (a) $r = 1 - \cos\theta$

   (b) $r = \cos 5\theta$

   (c) $r^2\theta = 1$

   (d) $r = 3 + 4\cos\theta$

5. Show that the curve $r = 2 - \csc\theta$ has the line $y = -1$ as a horizontal asymptote by showing that $\lim_{r\to\pm\infty} y = -1$. Use this fact to help sketch the curve.

6. Sketch the curve $(x^2 + y^2)^3 = 4x^2y^2$.