Chapter 12: Written problems

12.1 Three-dimensional Co-ordinate Systems

1. Find an equation of the sphere with center $(2, -6, 4)$ and radius 5. Describe its intersection with each of the co-ordinate planes.

2. Show that the equation represents a sphere, and find its center and radius.

$$x^2 + y^2 + z^2 + 8x - 6y + 2z + 17 = 0$$

3. Find an equation of a sphere if one of its diameters has endpoints $(2, 1, 4)$ and $(4, 3, 10)$.

4. Describe in words the region of $\mathbb{R}^3$ represented by the equations or inequalities.

   (a) $z^2 = 1$
   (b) $y^2 + z^2 = 16$
   (c) $x = z$
   (d) $x^2 + y^2 + z^2 > 2z$

5. Write inequalities to describe the solid cylinder that lies on or below the plane $z = 8$, and on or above the disk in the xy-plane with center the origin and radius 2.

12.2 Vectors

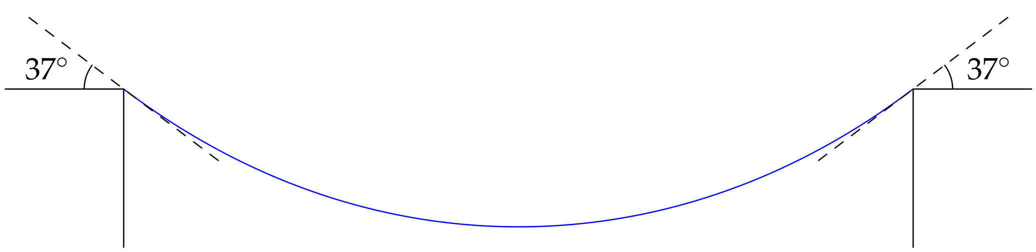
1. Find a vector **a** with representation given by the directed line segment $\overrightarrow{AB}$, where $A = (4, 0, -2)$ and $B = (4, 2, 1)$. Draw $\overrightarrow{AB}$ and the equivalent representation starting at the origin.

2. If $\mathbf{a} = 2\mathbf{i} - 4\mathbf{j} + 4\mathbf{k}$ and $\mathbf{b} = 2\mathbf{j} - \mathbf{k}$, find $\mathbf{a} + \mathbf{b}$, $2\mathbf{a} + 3\mathbf{b}$, $|\mathbf{a}|$, and $|\mathbf{a} - \mathbf{b}|$.

3. Find a vector with the same direction as $-2\mathbf{i} + 4\mathbf{j} + 2\mathbf{k}$ but with length 6.

4. If a child pulls a sled through the snow on a level path with a force of 50 N exerted at an angle of 38° above the horizontal, find the horizontal and vertical components of the force.

5. The tension **T** at either end of the chain has magnitude 25 N. What is the weight of the chain?



6. (a) Find the unit vectors that are parallel to the tangent line to the curve $y = 2\sin x$ at the point $(\pi/6, 1)$.
   (b) Find the unit vectors that are perpendicular to the tangent line.
   (c) Sketch the curve $y = 2\sin x$ and the vectors in parts (a) and (b), all starting at $(\pi/6, 1)$.

12.3 The Dot Product

1. Find the acute angle between the lines $x + 2y = 7$ and $5x - y = 2$.
2. Find the acute angles between the curves $y = \sin x$ and $y = \cos x$ at their point of intersection in the interval $(0, \pi/2)$. (The angle between two curves is the angle between their tangent lines at the point of intersection).
3. Find the scalar and vector projections of $\mathbf{b} = 5\mathbf{i} - \mathbf{j} + 4\mathbf{k}$ onto $\mathbf{a} = -2\mathbf{i} + 3\mathbf{j} - 6\mathbf{k}$. (Only vector projection examinable.)
4. A tow truck drags a stalled car along a road. The chain makes an angle of $30°$ with the road and the tension in the chain is 1500 N. How much work is done by the truck in pulling the car 1 km?
5. A boat sails south with the help of a wind blowing in the direction S36°E with magnitude 400 lb. Find the work done by the wind as the boat moves 120 ft.
6. Find the angle between a diagonal of a cube and a diagonal of one of its faces.

12.4 The Cross Product

1. If $\mathbf{a} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$, and $\mathbf{c} = \begin{pmatrix} 0 \\ 1 \\ 3 \end{pmatrix}$, show that $\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) \neq (\mathbf{a} \times \mathbf{b}) \times \mathbf{c}$
2. Find two unit vectors orthogonal to both $\mathbf{j} - \mathbf{k}$ and $\mathbf{i} + \mathbf{j}$.
3. Show that $(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{b} = 0$ for all vectors $\mathbf{a}, \mathbf{b}$ in $\mathbb{R}^3$.
4. Use the scalar triple product to determine whether the points $A(1, 3, 2)$, $B(3, -1, 6)$, $C(5, 2, 0)$, and $D(3, 6, -4)$ lie in the same plane.
5. (a) Find all vectors $\mathbf{v}$ that satisfy

$$\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} \times \mathbf{v} = \begin{pmatrix} 3 \\ 1 \\ -5 \end{pmatrix}$$

   (b) Explain why there is no vector $\mathbf{v}$ that satisfies

$$\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} \times \mathbf{v} = \begin{pmatrix} 3 \\ 1 \\ 5 \end{pmatrix}$$

12.5 Equations of Lines and Planes

1. Find an equation of the line.
   (a) The line through the origin and the point $(4, 3, -1)$.
   (b) The line through the points $(1.0, 2.4, 4.6)$ and $(2.6, 1.2, 0.3)$.
   (c) The line through $(2, 1, 0)$ and perpendicular to both $\mathbf{i} + \mathbf{j}$ and $\mathbf{j} + \mathbf{k}$.
   (d) The line of intersection of the planes $x + 2y + 3z = 1$ and $x - y + z = 1$.

2. Find an equation of the plane.

   (a) The plane through the point $(5, 3, 5)$ and with normal vector $2\mathbf{i} + \mathbf{j} - \mathbf{k}$.
   (b) The plane through the point $(2, 4, 6)$ and parallel to the plane $z = x + y$.
   (c) The plane through the origin and the points $(2, -4, 6)$ and $(5, 1, 3)$.
   (d) The plane that passes through the point $(1, 2, 3)$ and contains the line $x = 3t$, $y = 1 + t$, $z = 2 - t$.

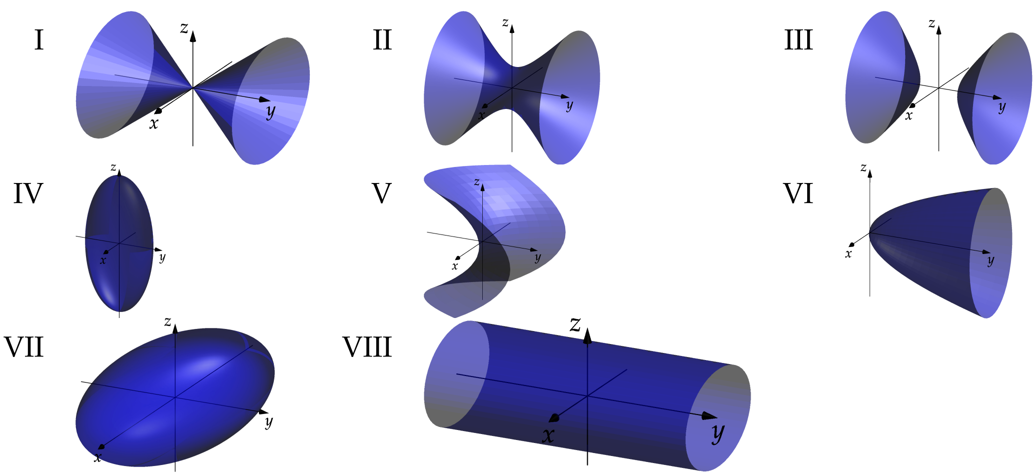
3. Find the point where the line $x = 1 + 2t$, $y = 4t$, $z = 2 - 3t$ intersects the plane $x + 2y - z = -1$.

4. Find an equation for the plane containing all points that are equidistant from the points $(2, 5, 5)$ and $(-6, 3, 1)$.

12.6 Cylinders and Quadrics

1. Match the equation with its graph (labeled I–VIII). Give reasons for your choices.

   (a) $x^2 + 4y^2 + 9z^2 = 1$
   (b) $9x^2 + 4y^2 + z^2 = 1$
   (c) $x^2 - y^2 + z^2 = 1$
   (d) $-x^2 + y^2 - z^2 = 1$
   (e) $y = 2x^2 + z^2$
   (f) $y^2 = x^2 + 2z^2$
   (g) $x^2 + 2z^2 = 1$
   (h) $y = x^2 - z^2$



2. Find an equation for the surface obtained by rotating the line $x = 3y$ about the x-axis.

3. Find an equation for the surface consisting of all points P for which the distance from P to the x-axis is twice the distance from P to the yz-plane. Identify the surface.

4. Show that the curve of intersection of the surfaces $x^2 + 2y^2 - z^2 + 3x = 1$ and $2x^2 + 4y^2 - 2z^2 - 5y = 0$ lies in a plane.