# Chapter 13: Written problems

## 13.1 Vector Functions and Spacecurves

1. At what points does the helix $\mathbf{r}(t) = \begin{pmatrix} \sin t \\ \cos t \\ t \end{pmatrix}$ intersect the sphere $x^2 + y^2 + z^2 = 5$?
2. Find a vector function that represents the curve of intersection of the two surfaces.
   (a) The cylinder $x^2 + y^2 = 4$ and the surface $z = xy$.
   (b) The paraboloid $z = 4x^2 + y^2$ and the parabolic cylinder $y = x^2$.
   (c) The semi-ellipsoid $x^2 + y^2 + 4z^2 = 4$, $y \geq 0$, and the cylinder $x^2 + z^2 = 1$.

## 13.2 Derivatives and Integrals of Vector Functions

1. Find the derivative of the vector valued function
   $$\mathbf{r}(t) = at\cos 3t\mathbf{i} + b\sin^3 t\mathbf{j} + c\cos^3 t\mathbf{k}$$
2. Find the unit tangent vector $\mathbf{T}(t)$ at the point on the curve
   $$\mathbf{r}(t) = (t^3 + 3t)\mathbf{i} + (t^2 + 1)\mathbf{j} + (3t + 4)\mathbf{k}$$
   where $t = 1$.
3. Find a point on the curve
   $$\mathbf{r}(t) = \begin{pmatrix} 2\cos t \\ 2\sin t \\ e^t \end{pmatrix}, \quad 0 \leq t \leq \pi$$
   where the tangent line is parallel to the plane $\sqrt{3}x + y = 1$.
4. Find $\mathbf{r}(t)$ if $\mathbf{r}'(t) = t\mathbf{i} + e^t\mathbf{j} + te^t\mathbf{k}$ and $\mathbf{r}(0) = \mathbf{i} + \mathbf{j} + \mathbf{k}$.

## 13.3 Arc-length and Curvature

1. Find the length of the curve $\mathbf{r}(t) = \cos t\mathbf{i} + \sin t\mathbf{j} + \ln\cos t\mathbf{k}$, $0 \leq t \leq \frac{\pi}{4}$.
2. Find the curvature of $\mathbf{r}(t) = t^2\mathbf{i} + \ln t\mathbf{j} + t\ln t\mathbf{k}$, at the point $(1, 0, 0)$.
3. At what point does the curve $y = \ln x$ have maximum curvature?

## 13.4 Motion in Space: Velocity and Acceleration

1. If a particle follows the path $\mathbf{r}(t) = e^t\mathbf{i} + e^{2t}\mathbf{j}$, find its velocity, acceleration, and speed. Sketch the path and draw the velocity and acceleration vectors when $t = 0$.
2. Show that if a particle travels at constant speed, then its velocity and acceleration vectors are orthogonal.
3. A gun is fired with angle of elevation 30°. What is the muzzle speed if the maximum height of the shell is 500 m?