Math 2D Multi-Variable Calculus Homework Questions 1

10 Parametric Equations and Polar Co-ordinates

10.1 Curves Defined by Parametric Equations

2–4 Sketch the curve by using the parametric equations to plot points. Indicate with an arrow the direction in which the curve is traced as t increases.

2. $x = t^2, \quad y = t^3 - 4t, \quad -3 \le t \le 3$

4. $x = e^{-t} + t, \quad y = e^t - t, \quad -2 \le t \le 2$

6–16 (a) Sketch the curve parameterized by x and y, indicating the direction of travel. When the domain of the parameter is not given, assume that it is as large as possible.

(b) Eliminate the parameter to find a Cartesian equation of the curve.

6. $x = 1 - 2t, \quad y = \frac{1}{2}t - 1, \quad -2 \le t \le 4.$

8. * $x = t - 1, \quad y = t^3 + 1, \quad -2 \le t \le 2.$

12. $x = \frac{1}{2}\cos\theta, \quad y = 2\sin\theta, \quad 0 \le \theta \le \pi.$

16. $x = \sqrt{t+1}, \quad y = \sqrt{t-1}.$

28. * Match the parametric equations with the graphs labeled I–VI. Give reasons for your choice. *Don't even think about using a graphing device for this!*

(a) $x = t^4 - t + 1, \quad y = t^2.$

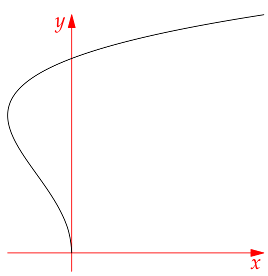
(b) $x = t^2 - 2t, \quad y = \sqrt{t}.$

(c) $x = \sin 2t, \quad y = \sin(t + \sin 2t).$

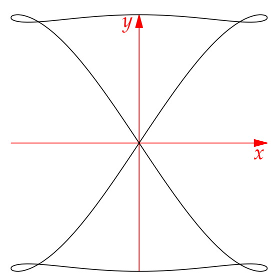
(d) $x = \cos 5t, \quad y = \sin 2t.$

(e) $x = t + \sin 4t, \quad y = t^2 + \cos 3t.$

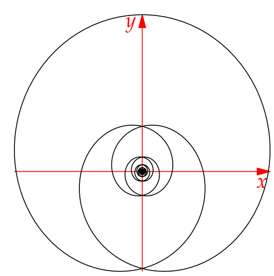
(f) $x = \dfrac{\sin 2t}{4 + t^2}, \quad y = \dfrac{\cos 2t}{4 + t^2}.$



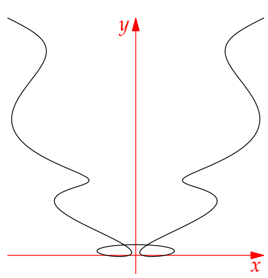
I



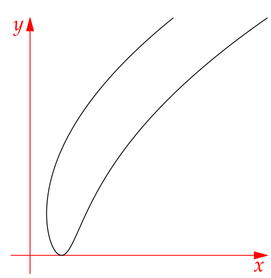
II



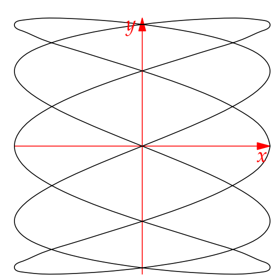
III



IV



V



VI

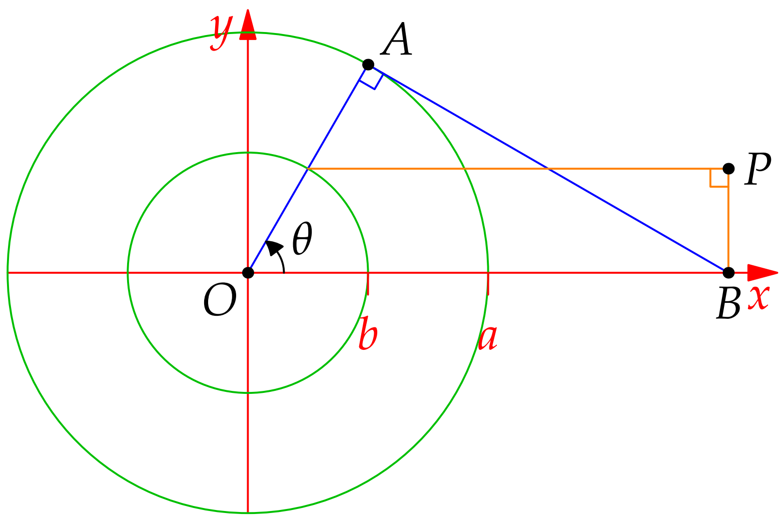
40. Let P be a point at a distance d from the center of a circle of radius r. The curve traced out by P as the circle rolls along the straight line is called a *trochoid*. The cycloid is the special case where $d = r$. Supposing that $t = 0$ when P is at its lowest point, where t is the same angle used in our description of the cycloid, prove that the parametric equations of the trochoid are

$$x(t) = rt - d\sin t, \qquad y = r - d\cos t.$$

42. If a and b are fixed numbers, find parametric equations for the curve that consists of all possible positions of the point P in the figure, using the angle θ as the parameter. The line segment AB is tangent to the larger circle. Show further, by eliminating the parameter, that the Cartesian equation of the curve is

$$\frac{a^2}{x^2} + \frac{y^2}{b^2} = 1.$$

Finally, sketch the curve!



10.2 Calculus with Parametric Curves

4. Find an equation of the tangent line to the curve parametrized by $x = t - t^{-1}$ and $y = 1 + t^2$, at $t = 1$.

6. * Repeat question 4. for $x = \sin^3\theta, \quad y = \cos^3\theta, \quad \theta = \frac{\pi}{6}$.

10. A curve is parameterized by $x = \cos t + \cos 2t, \quad y = \sin t + \sin 2t$. Find the equation of the tangent(s) to the curve at the point $(-1, 1)$. Also graph the curve and the tangents.

12. * Let $x = t^3 + 1, \quad y = t^2 - t$. Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ and $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$. For which values of t is the curve concave up?

18. A curve is parameterized by $x = t^3 - 3t, \quad y = t^3 - 3t^2$. Find the points on the curve where the tangent is horizontal or vertical.

30. Find equations of the tangents to the curve $x = 3t^2 + 1, \quad y = 2t^3 + 1$ that pass through the point $(4, 3)$.

32. Find the area enclosed by the curve $x = t^2 - 2t, \quad y = \sqrt{t}$ and the y-axis.

42. Find the exact length of the curve defined by $x = e^t + e^{-t}, \quad y = 5 - 2t$, where $0 \le t \le 3$.

44. * Find the exact length of the curve defined by $x = 3\cos t - \cos 3t, \quad y = 3\sin t - \sin 3t$, where $0 \le t \le \pi$.

10.3 Polar Co-ordinates

8–12 Sketch the region in the plane consisting of points whose polar co-ordinates satisfy the given conditions.

8. $0 \le r < 2, \quad \pi \le \theta \le \frac{3\pi}{2}$

10. $1 \le r \le 3, \quad \frac{\pi}{6} < \theta < \frac{5\pi}{6}$

12. $r \ge 1, \quad \pi \le \theta \le 2\pi$

16–20 Identify the curve by finding a Cartesian equation for the curve.

16. $r = 4\sec\theta$

20. * $r = \tan\theta\sec\theta$

22–26 Find a polar equation for the curve represented by the given Cartesian equation.

22. $y = x$

26. * $xy = 4$

30–46 Sketch the curve with the given polar equation.

30. $r = 1 - \cos\theta$

36. * $r = \cos 5\theta$

44. $r^2\theta = 1$

46. $r = 3 + 4\cos\theta$

50. Show that the curve $r = 2 - \csc\theta$ has the line $y = -1$ as a horizontal asymptote by showing that $\lim\limits_{r\to\pm\infty} y = -1$. Use this fact to help sketch the curve.

52. Sketch the curve $(x^2 + y^2)^3 = 4x^2y^2$.