Math 2D Multi-Variable Calculus Homework Questions 2

12 Vectors and the Geometry of Space

12.1 Three-dimensional Co-ordinate Systems

12 Find an equation of the sphere with center $(2, -6, 4)$ and radius 5. Describe its intersection with each of the co-ordinate planes.

16 * Show that the equation represents a sphere, and find its center and radius.

$$x^2 + y^2 + z^2 + 8x - 6y + 2z + 17 = 0$$

20 * Find an equation of a sphere if one of its diameters has endpoints $(2, 1, 4)$ and $(4, 3, 10)$.

28–34 Describe in words the region of $\mathbb{R}^3$ represented by the equations or inequalities.

28 $z^2 = 1$

30 $y^2 + z^2 = 16$

32 $x = z$

34 $x^2 + y^2 + z^2 > 2z$

36 Write inequalities to describe the solid cylinder that lies on or below the plane $z = 8$, and on or above the disk in the xy-plane with center the origin and radius 2.

12.2 Vectors

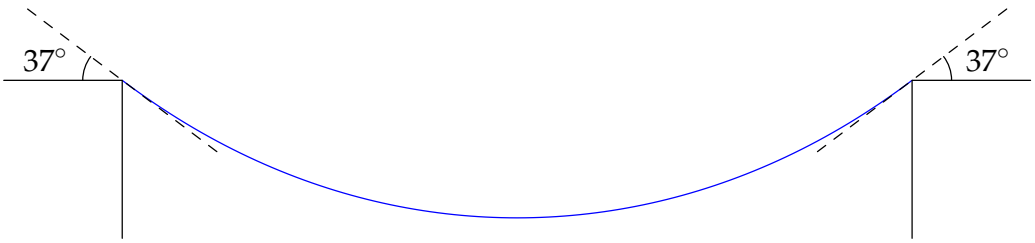
14 Find a vector $\mathbf{a}$ with representation given by the directed line segment $\overrightarrow{AB}$, where $A = (4, 0, -2)$ and $B = (4, 2, 1)$. Draw $\overrightarrow{AB}$ and the equivalent representation starting at the origin.

22 If $\mathbf{a} = 2\mathbf{i} - 4\mathbf{j} + 4\mathbf{k}$ and $\mathbf{b} = 2\mathbf{j} - \mathbf{k}$, find $\mathbf{a} + \mathbf{b}$, $2\mathbf{a} + 3\mathbf{b}$, $|\mathbf{a}|$, and $|\mathbf{a} - \mathbf{b}|$.

26 Find a vector with the same direction as $-2\mathbf{i} + 4\mathbf{j} + 2\mathbf{k}$ but with length 6.

30 * If a child pulls a sled through the snow on a level path with a force of 50 N exerted at an angle of 38° above the horizontal, find the horizontal and vertical components of the force.

38 * The tension $\mathbf{T}$ at either end of the chain has magnitude 25 N. What is the weight of the chain?



42 (a) Find the unit vectors that are parallel to the tangent line to the curve $y = 2\sin x$ at the point $(\pi/6, 1)$.

(b) Find the unit vectors that are perpendicular to the tangent line.

(c) Sketch the curve $y = 2\sin x$ and the vectors in parts (a) and (b), all starting at $(\pi/6, 1)$.

12.3 The Dot Product

30 Find the acute angle betweent the lines $x + 2y = 7$ and $5x - y = 2$.

32 Find the acute angles between the curves $y = \sin x$ and $y = \cos x$ at their point of intersection in the interval $(0, \pi/2)$. (The angle between two curves is the angle between their tangent lines at the point of intersection).

42 Find the scalar and vector projections of $\mathbf{b} = 5\mathbf{i} - \mathbf{j} + 4\mathbf{k}$ onto $\mathbf{a} = -2\mathbf{i} + 3\mathbf{j} - 6\mathbf{k}$. (Only vector projection examinable.)

50 A tow truck drags a stalled car along a road. The chain makes an angle of 30° with the road and the tension in the chain is 1500 N. How much work is done by the truck in pulling the car 1 km?

52 * A boat sails south with the help of a wind blowing in the direction S36°E with magnitude 400 lb. Find the work done by the wind as the boat moves 120 ft.

56 * Find the angle between a diagonal of a cube and a diagonal of one of its faces.

12.4 The Cross Product

18 If $\mathbf{a} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$, and $\mathbf{c} = \begin{pmatrix} 0 \\ 1 \\ 3 \end{pmatrix}$, show that

$$\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) \neq (\mathbf{a} \times \mathbf{b}) \times \mathbf{c}.$$

20 Find two unit vectors orthogonal to both $\mathbf{j} - \mathbf{k}$ and $\mathbf{i} + \mathbf{j}$.

22 Show that $(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{b} = 0$ for all bectors $\mathbf{a}, \mathbf{b}$ in $\mathbb{R}^3$.

38 * Use the scalar triple product to determine whether the points $A(1, 3, 2)$, $B(3, -1, 6)$, $C(5, 2, 0)$, and $D(3, 6, -4)$ lie in the same plane.

44 *(a) Find all vectors $\mathbf{v}$ that satisfy

$$\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} \times \mathbf{v} = \begin{pmatrix} 3 \\ 1 \\ -5 \end{pmatrix}$$

(b) Explain why there is no vector $\mathbf{v}$ that satisfies

$$\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} \times \mathbf{v} = \begin{pmatrix} 3 \\ 1 \\ 5 \end{pmatrix}$$