Math 2D Multi-Variable Calculus Homework Questions 3

12.5 Equations of Lines and Planes

6–12 Find an equation of the line.

6 The line through the origin and the point $(4, 3, -1)$.

8 The line through the points $(1.0, 2.4, 4.6)$ and $(2.6, 1.2, 0.3)$.

10 * The line through $(2, 1, 0)$ and perpendicular to both $\mathbf{i} + \mathbf{j}$ and $\mathbf{j} + \mathbf{k}$.

12 * The line of intersection of the planes $x + 2y + 3z = 1$ and $x - y + z = 1$.

24–34 Find an equation of the plane.

24 * The plane through the point $(5, 3, 5)$ and with normal vector $2\mathbf{i} + \mathbf{j} - \mathbf{k}$.

28 The plane through the point $(2, 4, 6)$ and parallel to the plane $z = x + y$.

32 * The plane through the origin and the points $(2, -4, 6)$ and $(5, 1, 3)$.

34 * The plane that passes through the point $(6, 0, -2)$ and contains the line $x = 3t$, $y = 1 + t$, $z = 7 + 4t$.

46 Find the point where the line $x = 1 + 2t$, $y = 4t$, $z = 2 - 3t$ intersects the plane $x + 2y - z = -1$.

63 Find an equation for the plane containing all points that are equidistant from the points $(2, 5, 5)$ and $(-6, 3, 1)$.

12.6 Cylinders and Quadrics

21–28 *(submit all 8) Match the equation with its graph (labeled I–VIII). Give reasons for your choices.

21 $x^2 + 4y^2 + 9z^2 = 1$

22 $9x^2 + 4y^2 + z^2 = 1$

23 $x^2 - y^2 + z^2 = 1$

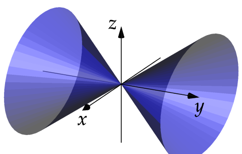
24 $-x^2 + y^2 - z^2 = 1$

25 $y = 2x^2 + z^2$

26 $y^2 = x^2 + 2z^2$

27 $x^2 + 2z^2 = 1$

28 $y = x^2 - z^2$

I


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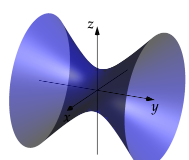

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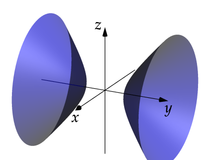

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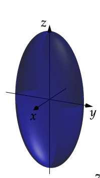

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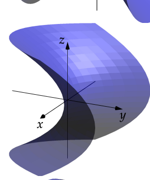

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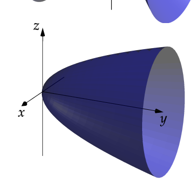

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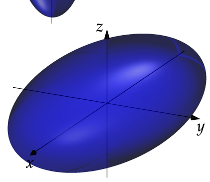

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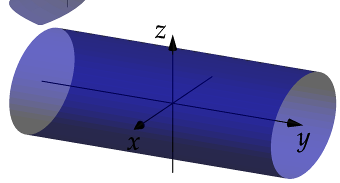

44 * Find an equation for the surface obtained by rotating the line $x = 3y$ about the y-axis. (Typo - book rotates about x-axis)

46 Find an equation for the surface consisting of all points P for which the distance from P to the x-axis is twice the distance from P to the yz-plane. Identify the surface.

50 * Show that the curve of intersection of the surfaces $x^2 + 2y^2 - z^2 + 3x = 1$ and $2x^2 + 4y^2 - 2z^2 - 5y = 0$ lies in a plane.

13 Vector Functions

13.1 Vector Functions and Spacecurves

30 * At what points does the helix $\mathbf{r}(t) = \begin{pmatrix} \sin t \\ \cos t \\ t \end{pmatrix}$ intersect the sphere $x^2 + y^2 + z^2 = 5$?

40–44 Find a vector function that represents the curve of intersection of the two surfaces.

40 The cylinder $x^2 + y^2 = 4$ and the surface $z = xy$.

42 * The paraboloid $z = 4x^2 + y^2$ and the parabolic cylinder $y = x^2$.

44 The semi-ellipsoid $x^2 + y^2 + 4z^2 = 4$, $y \geq 0$, and the cylinder $x^2 + z^2 = 1$.

13.2 Derivatives and Integrals of Vector Functions

14 Find the derivative of the vector valued function

$$\mathbf{r}(t) = at \cos 3t\mathbf{i} + b \sin^3 t\mathbf{j} + c \cos^3 t\mathbf{k}.$$

18 Find the unit tangent vector $\mathbf{T}(t)$ at the point on the curve

$$\mathbf{r}(t) = (t^3 + 3t)\mathbf{i} + (t^2 + 1)\mathbf{j} + (3t + 4)\mathbf{k}$$

where $t = 1$.

28 Find a point on the curve

$$\mathbf{r}(t) = \begin{pmatrix} 2\cos t \\ 2\sin t \\ e^t \end{pmatrix}, \quad 0 \leq t \leq \pi$$

where the tangent line is parallel to the plane $\sqrt{3}x + y = 1$.

42 * Find $\mathbf{r}(t)$ if $\mathbf{r}'(t) = t\mathbf{i} + e^t\mathbf{j} + te^t\mathbf{k}$ and $\mathbf{r}(0) = \mathbf{i} + \mathbf{j} + \mathbf{k}$.

13.3 Arc-length and Curvature

4 Find the length of the curve $\mathbf{r}(t) = \cos t\mathbf{i} + \sin t\mathbf{j} + \ln \cos t\mathbf{k}$, $0 \leq t \leq \frac{\pi}{4}$.

24 Find the curvature of $\mathbf{r}(t) = t^2\mathbf{i} + \ln t\mathbf{j} + t \ln t\mathbf{k}$, at the point $(1, 0, 0)$.

30 At what point does the curve $y = \ln x$ have maximum curvature?

13.4 Motion in Space: Velocity and Acceleration

6 If a particle follows the path $\mathbf{r}(t) = e^t\mathbf{i} + e^{2t}\mathbf{j}$, find its velocity, acceleration, and speed. Sketch the path and draw the velocity and acceleration vectors when $t = 0$.

22 Show that if a particle travels at constant speed, then its velocity and acceleration vectors are orthogonal.

26 A gun is fired with angle of elevation 30°. What is the muzzle speed if the maximum height of the shell is 500 m?

14 Partial Derivatives

14.1 Functions of Several Variables

14–18 Find and sketch the domain of the function.

14 $f(x,y) = \sqrt{xy}$.

16 $f(x,y) = \sqrt{x^2 - y^2}$.

18 $f(x,y) = \sqrt{y} + \sqrt{25 - x^2 - y^2}$.

24–30 Sketch some contour lines for each function. Use these to help sketch the graph of the function.

24 $f(x,y) = 2 - x$.

26 $f(x,y) = e^{-y}$.

28 $f(x,y) = 1 + 2x^2 + 2y^2$.

30 $f(x,y) = \sqrt{4x^2 + y^2}$.

14.2 Limits and Continuity

30–36 Determine the set of points at which the function is continuous.

30 $F(x,y) = \cos\sqrt{1 + x - y}$.

32 $H(x,y) = \dfrac{e^x + e^y}{e^{xy} - 1}$.

34 $G(x,y) = \tan^{-1}\left((x^2 + y^2)^{-2}\right)$.

36 $f(x,y,z) = \sqrt{y - x^2}\ln z$.

14.3 Partial Derivatives

16–40 Find the first partial derivatives of the function.

16 $f(x,y) = x^4y^3 + 8x^2y$

20 $z = \tan xy$

24 $w = \dfrac{e^v}{u + v^2}$

28 $f(x,y) = x^y$

32 $f(x,y,z) = x \sin(y - z)$

36 $u = x^{y/z}$

40 $u = \sin(x_1 + 2x_2 + \cdots + nx_n)$

54–58 Find all the second partial derivatives.

54 $f(x,y) = \sin^2(mx + ny)$

56 $v = \dfrac{xy}{x - y}$

58 $v = e^{xe^y}$

84 Show that the Cobb-Douglas production function $P = bL^\alpha K^\beta$ satisfies the equation

$$L\frac{\partial P}{\partial L} + K\frac{\partial P}{\partial K} = (\alpha + \beta)P$$

90 If a, b, c are the sides of a triangle, and A, B, C are the opposite angles, find $\frac{\partial A}{\partial a}, \frac{\partial A}{\partial b}, \frac{\partial A}{\partial c}$ by implicit differentiation of the Law of Cosines.