Math 2D Multi-Variable Calculus Homework Questions 4

14.4 Tangent Planes and Linear Approximations

2, 6 Find an equation of the tangent plane to the given surface at the specified point.

2 * $z = 3(x-1)^2 + 2(y+3)^2 + 7, \qquad (2, -2, 12)$

6 $z = \ln(x - 2y), \qquad (3, 1, 0)$

12 * Find the linearization of the function $f(x, y) = x^3y^4$ at the point $(1, 1)$.

18 * Verify the linear approximation at $(0, 0)$:

$$\sqrt{y + \cos^2 x} \approx 1 + \frac{1}{2}y$$

34 Use differentials to estimate the amount of metal in a closed cylindrical can that is 10 cm high and 4 cm in diameter if the metal in the top and bottom is 0.1 cm thick and the metal in the sides is 0.05 cm thick.

40 Four positive numbers, each less than 50, are rounded to the first decimal place and then multiplied together. Use differentials to estimate the maximum possible error in the computed product that might result from rounding.

42 * Suppose you need to know an equation of the tangent plane to a surface S at the point $P(2, 1, 3)$. You don't have an equation for S but you know that the curves

$$\mathbf{r}_1(t) = \begin{pmatrix} 2+3t \\ 1-t^2 \\ 3-4t+t^2 \end{pmatrix}, \quad \text{and} \quad \mathbf{r}_2(u) = \begin{pmatrix} 1+u^2 \\ 2u^3-1 \\ 2u+1 \end{pmatrix}$$

both lie on S. Find an equation of the tangent plane at P.

14.5 The Chain Rule

2 Use the chain rule to find $\frac{dz}{dt}$ if

$$z = \cos(x + 4y), \qquad x = 5t^4, \qquad y = \frac{1}{t}$$

Then, substitute each of x and y into the expression for z and compute the derivative using single variable calculus.

8 * Use the chain rule to find $\frac{\partial z}{\partial s}$ and $\frac{\partial z}{\partial t}$ if

$$z = \arcsin(x - y), \qquad x = s^2 + t^2, \qquad y = 1 - 2st$$

Now substitute and compute as in question 2.

18 State the chain rule for the derivative $\frac{\partial R}{\partial v}$ if

$$R = f(x, y, z, t), \quad x = x(u, v, w), \quad y = y(u, v, w), \quad z = z(u, v, w), \quad t = t(u, v, w).$$

26 * Suppose that

$$u = xe^{ty}, \qquad x = \alpha^2\beta, \qquad y = \beta^2\gamma, \qquad t = \gamma^2\alpha.$$

Use the chain rule to compute the derivatives $\frac{\partial u}{\partial \alpha}, \frac{\partial u}{\partial \beta}, \frac{\partial u}{\partial \gamma}$ when $\alpha = -1,\ \beta = 2,\ \gamma = 1$.

36 * Wheat production W in a given year depends on the average temperatue T and the annual rainfall R. Scientists estimate that the average temperature is rising at a rate of $0.15°\mathrm{C/year}$ and rainfall is decreasing at a rate of $0.1\,\mathrm{cm/year}$. They also estimate that, at current production levels, $\frac{\partial W}{\partial T} = -2$ and $\frac{\partial W}{\partial R} = 8$.

(a) What is the significance of the signs of these partial derivatives?

(b) Estimate the current rate of change of wheat production, $\frac{\mathrm{d}W}{\mathrm{d}t}$.

42 A manufacturer has modeled its yearly production function P (the value of its entire production in millions of dollars) as a Cobb–Douglas function

$$P(L, K) = 1.47L^{0.65}K^{0.35}$$

where L is the number of labor hours (in thousands) and K is the invested capital (in millions of dollars). Suppose that when $L = 30$ and $K = 8$, the labor force is decreasing at a rate of 2000 labor hours per year and capital is increasing at a rate of \$500,000 per year. Find the rate of change of production.